

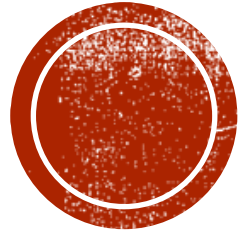
**INTRODUCTION TO
COMPUTATIONAL
TOPOLOGY**

**HSIEN-CHIH CHANG
LECTURE 14, OCTOBER 28, 2021**

ADMINISTRIVIA

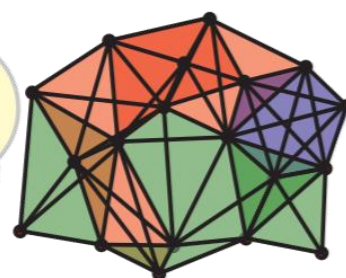
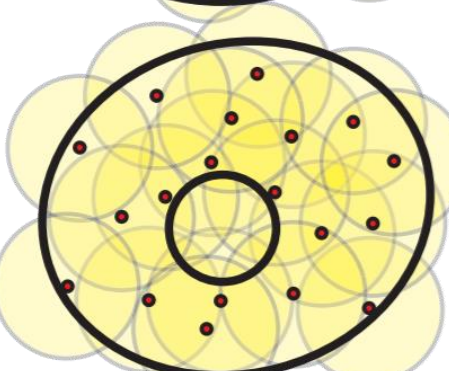
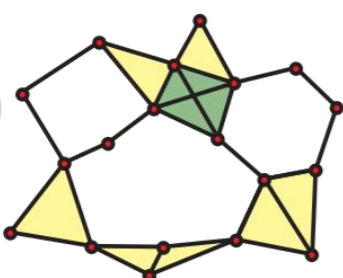
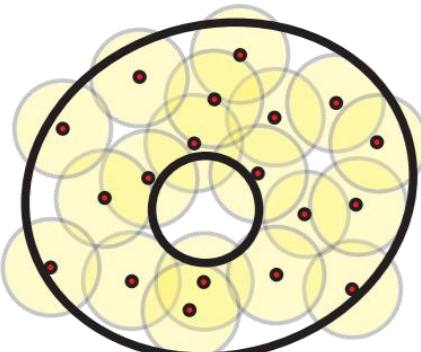
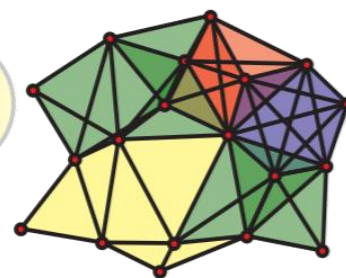
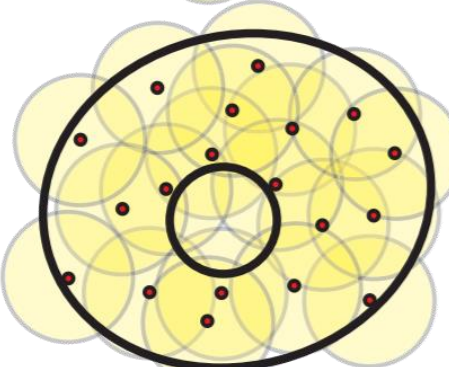
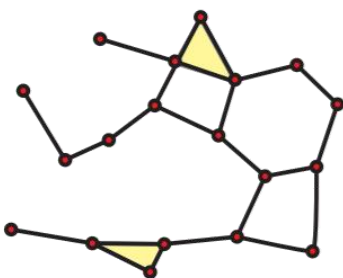
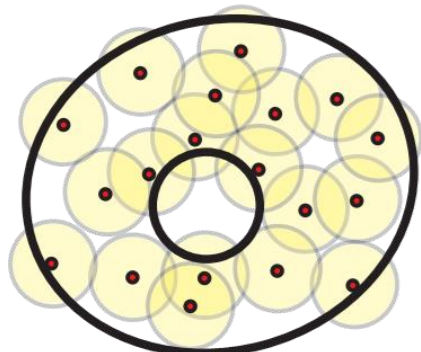
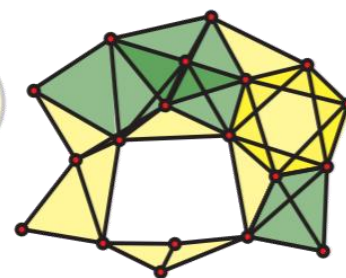
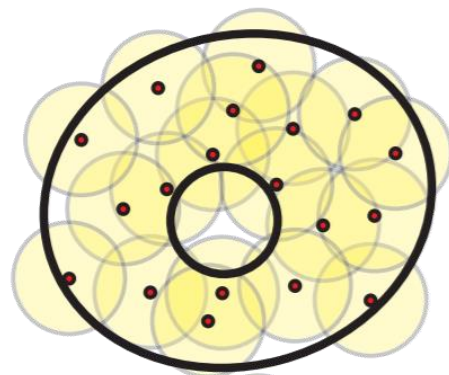
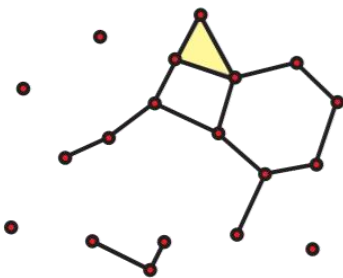
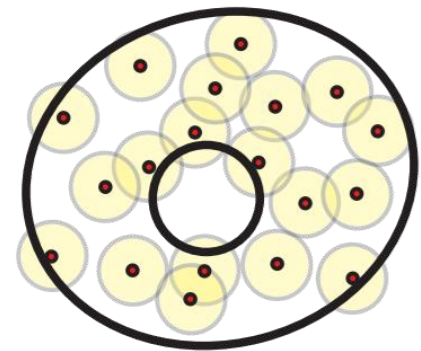
- Homework 4 is out, due 11/8 (Mon)
- Come talk to me about the project (as a group)





PERSISTENCE DIAGRAM



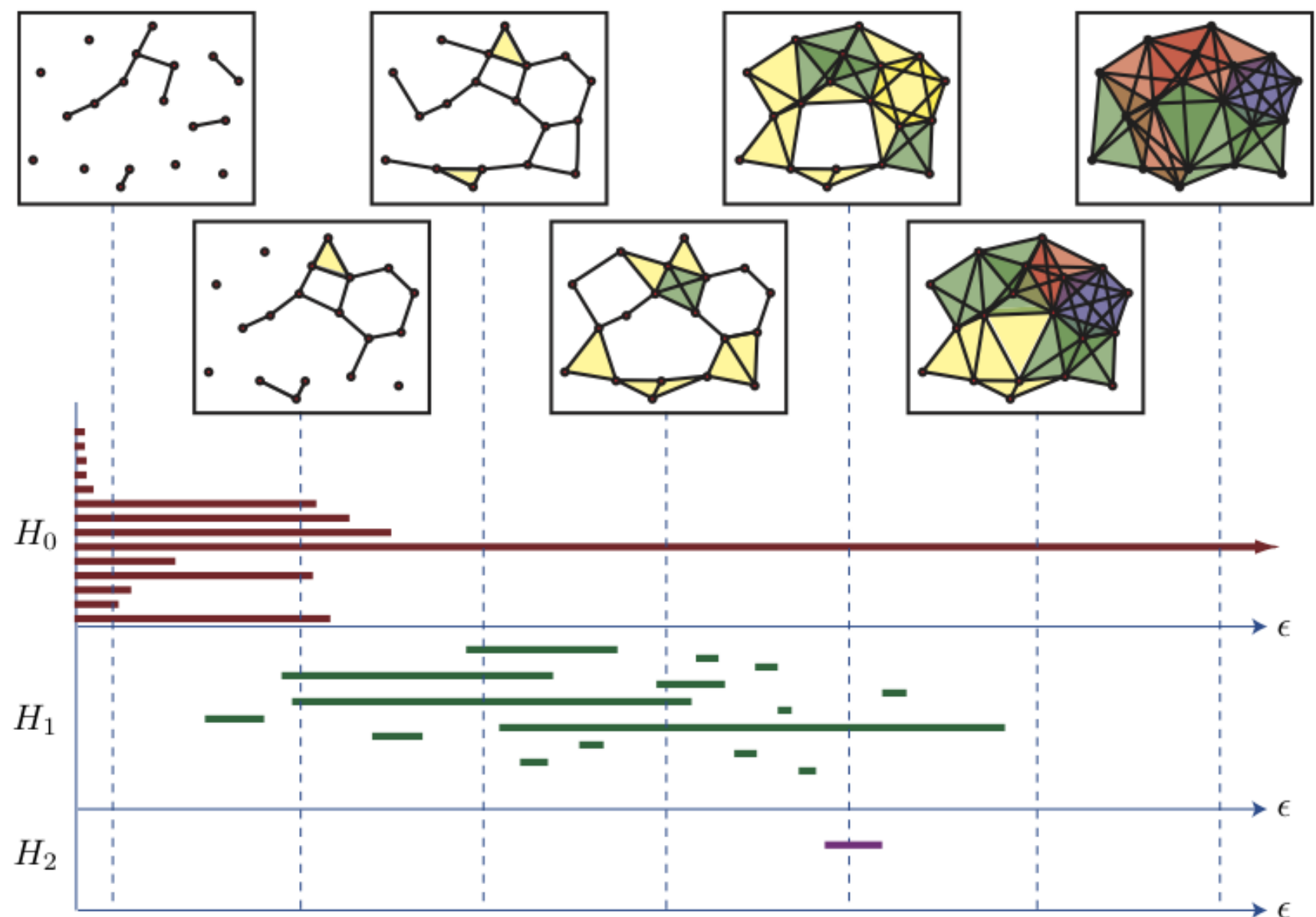


COOL IDEA

- Don't guess ϵ ; take all!

[Christ 2008]





PERSISTENCE

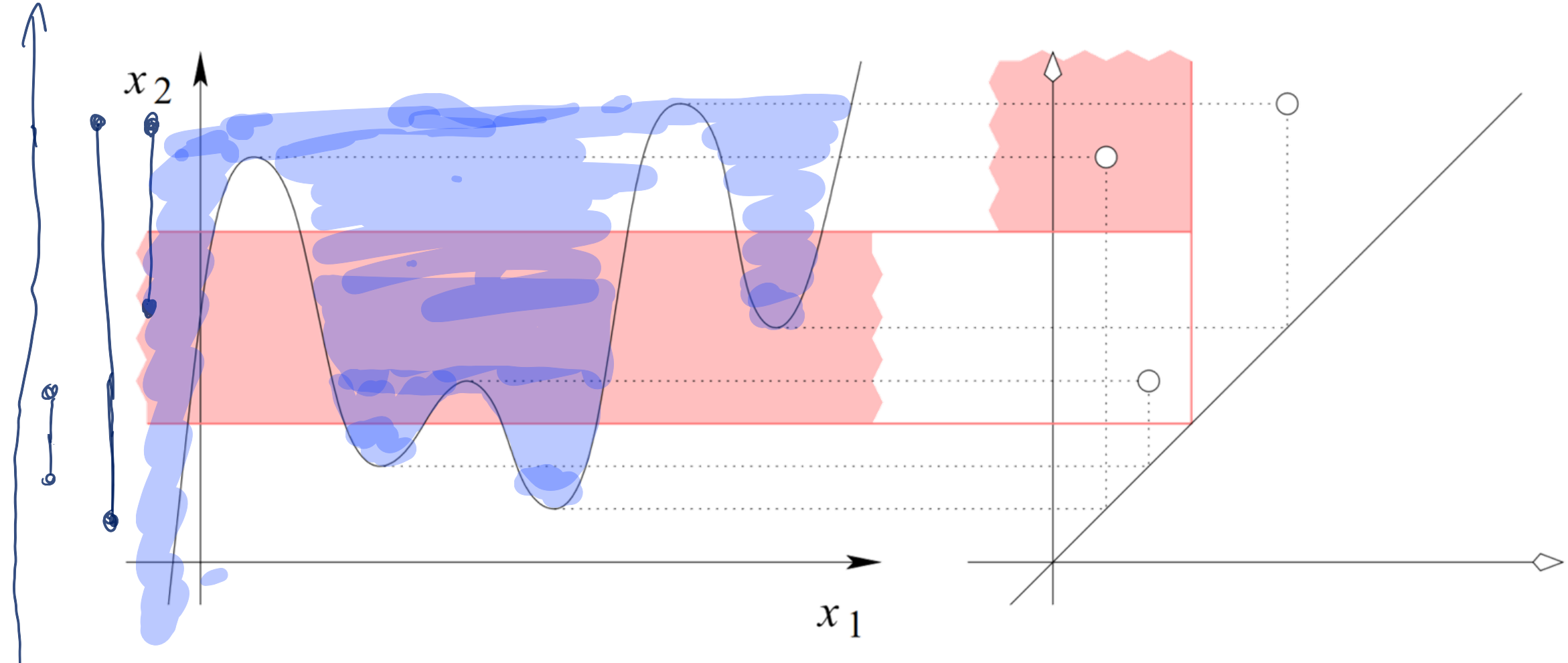
- $\text{Pers}[\gamma] = \epsilon_j - \epsilon_i$

[Christ 2008]



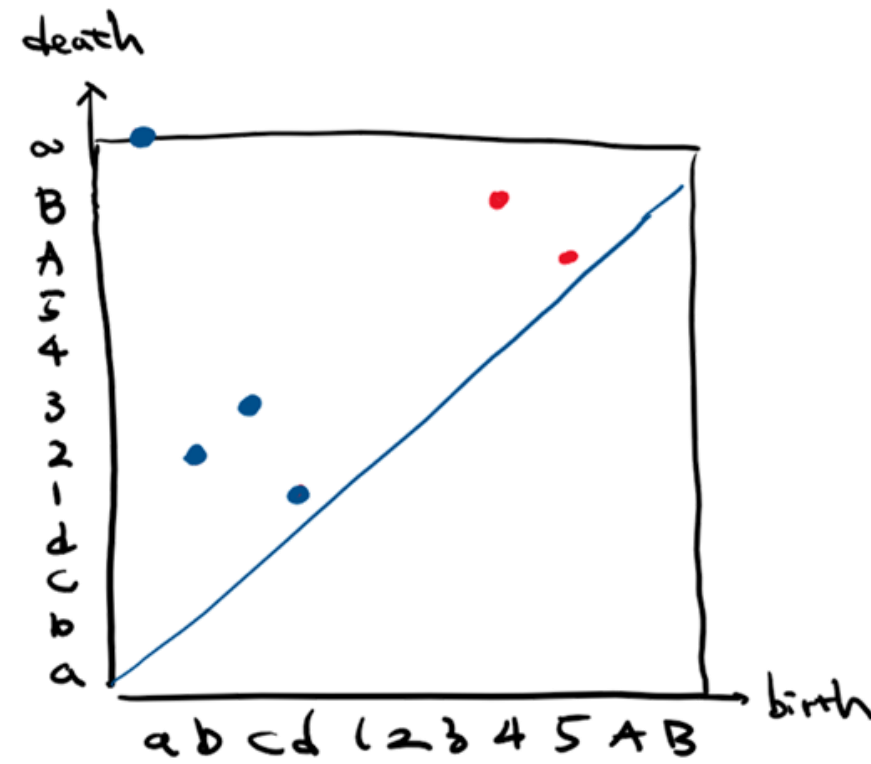
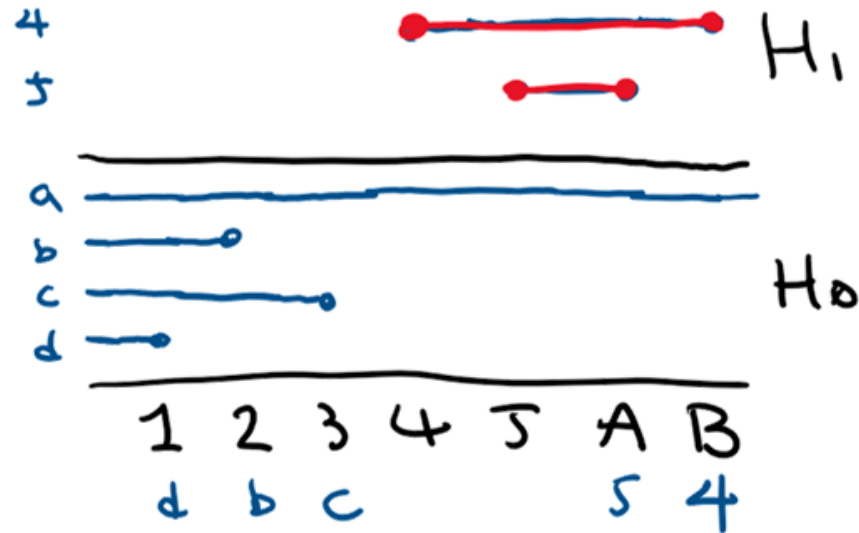
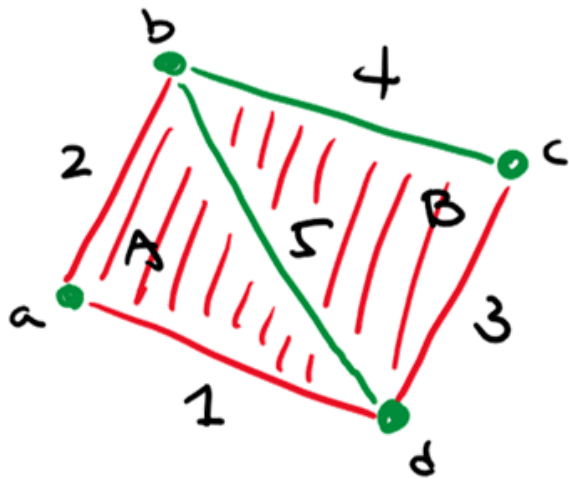
PERSISTENCE DIAGRAM

[CohenSteiner-Edelsbrunner-Harer 2005]

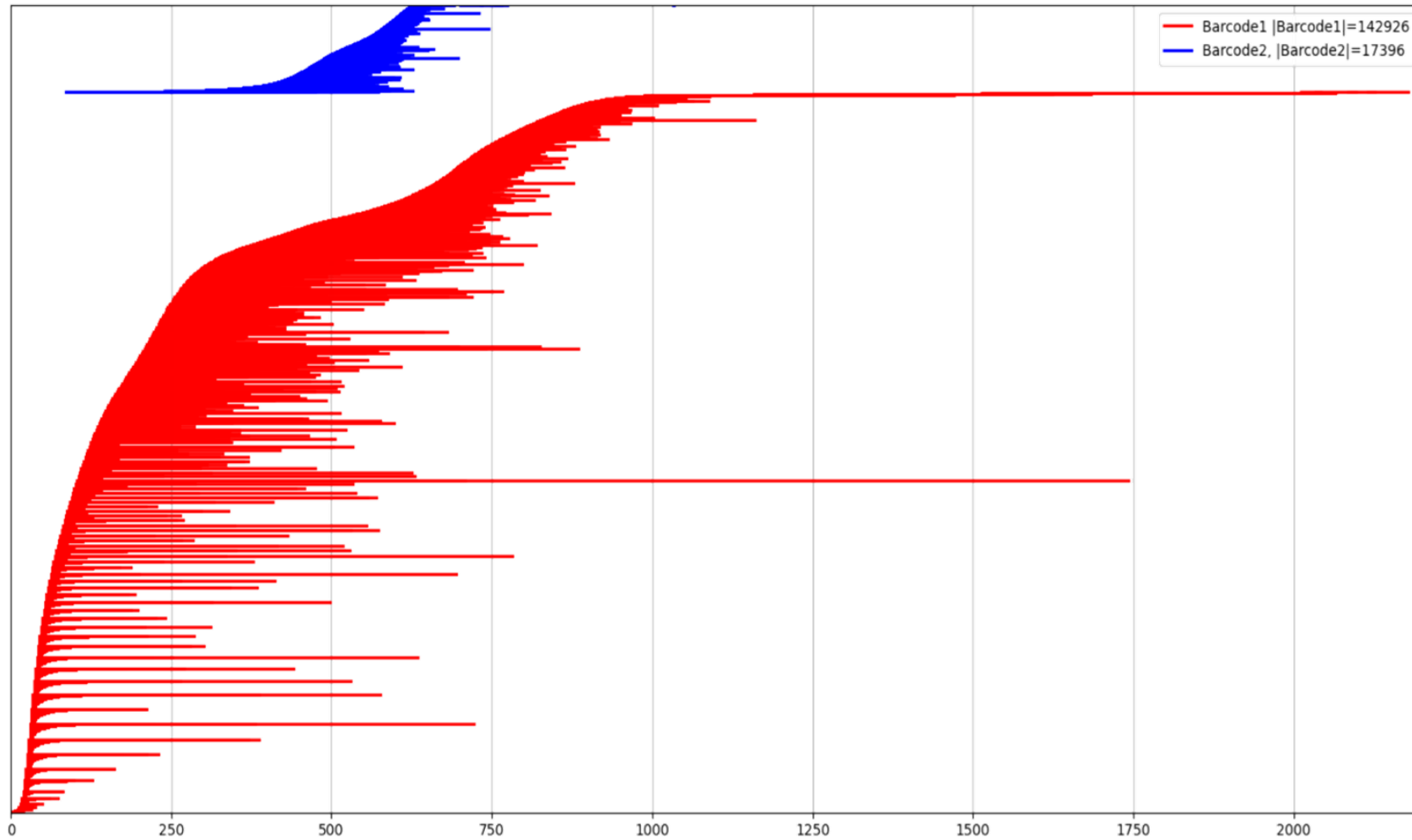


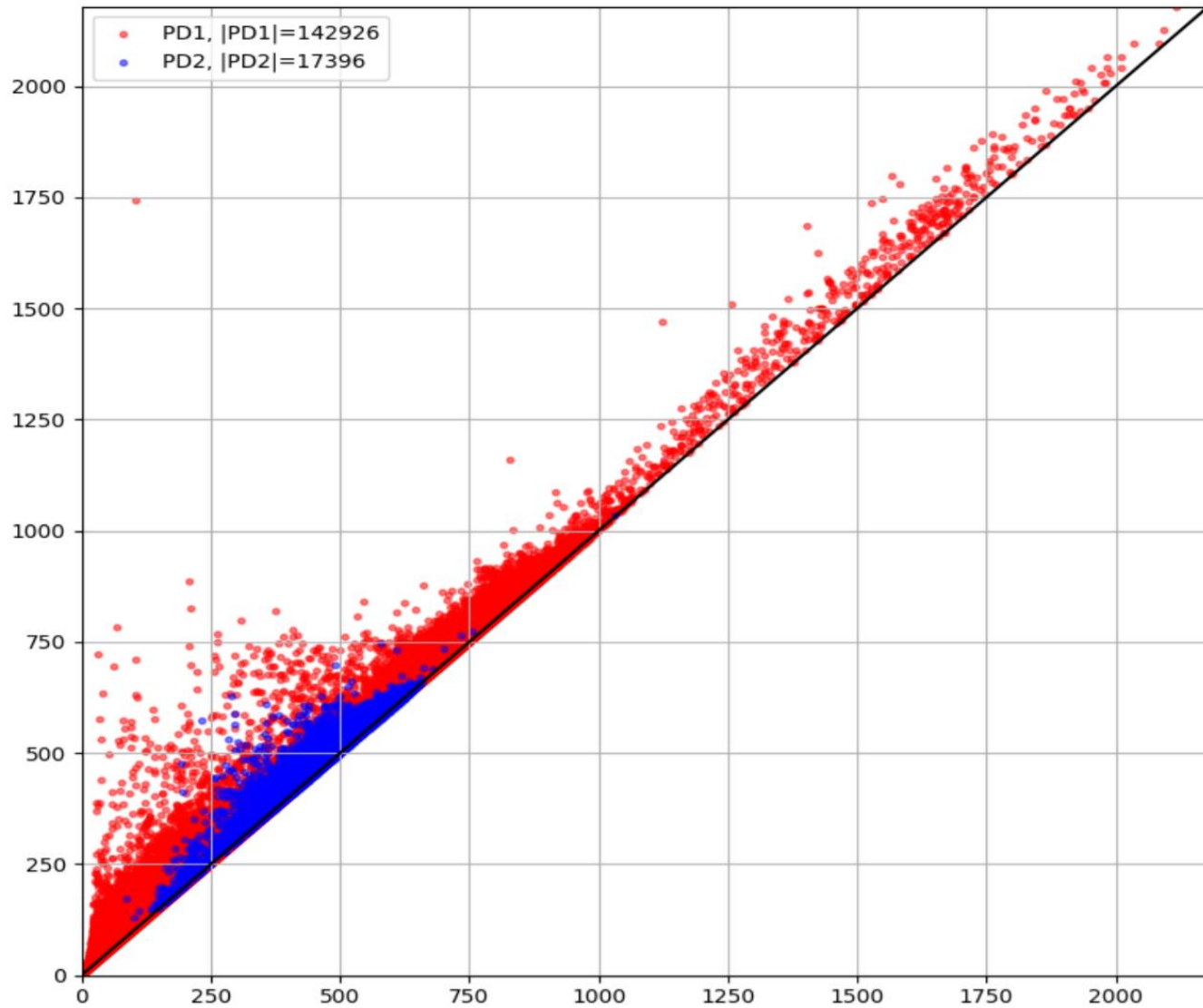
PERSISTENCE DIAGRAM

[CohenSteiner-Edelsbrunner-Harer 2005]



PERSISTENCE DIAGRAM





PERSISTENCE DIAGRAM

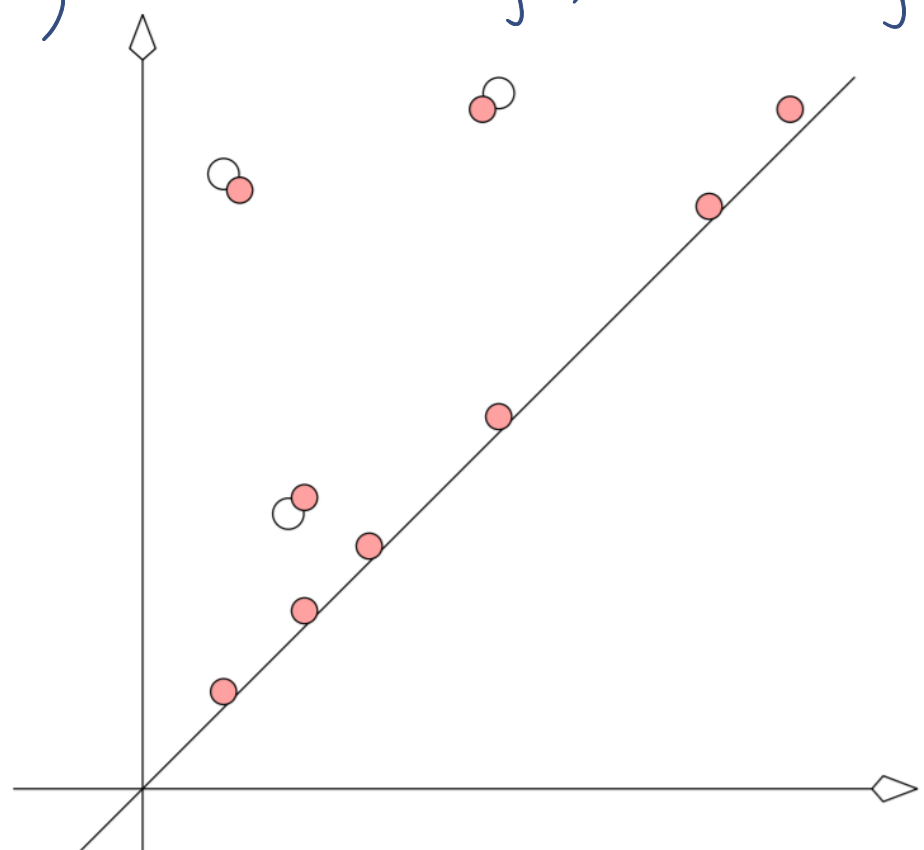
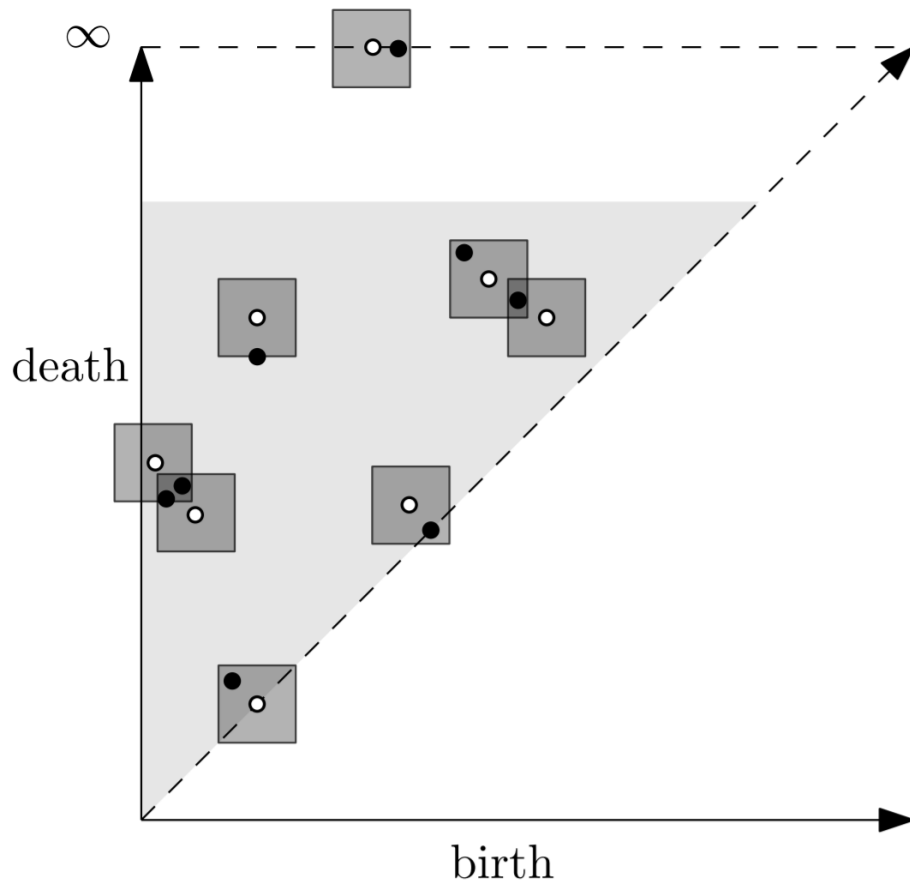


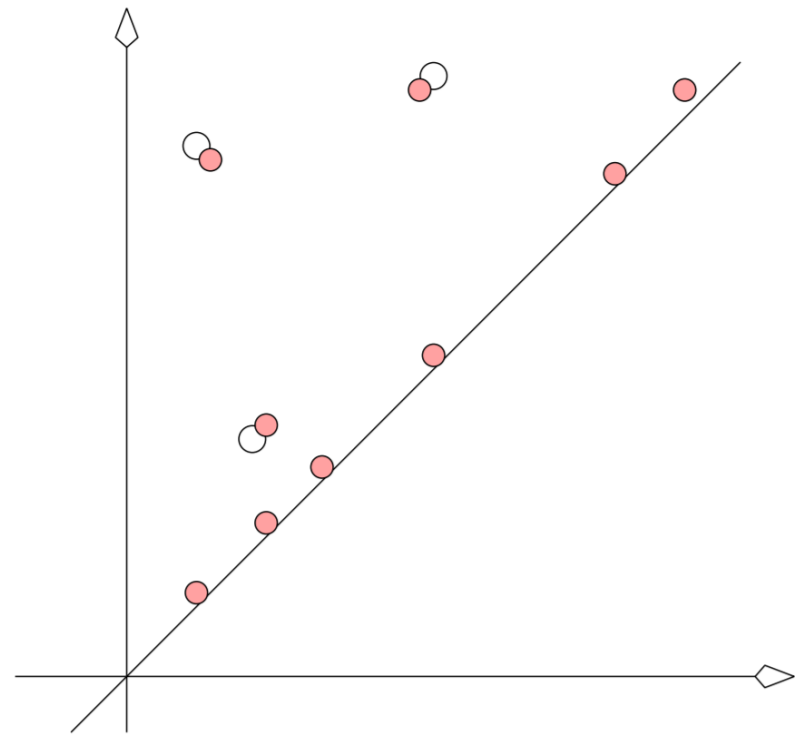
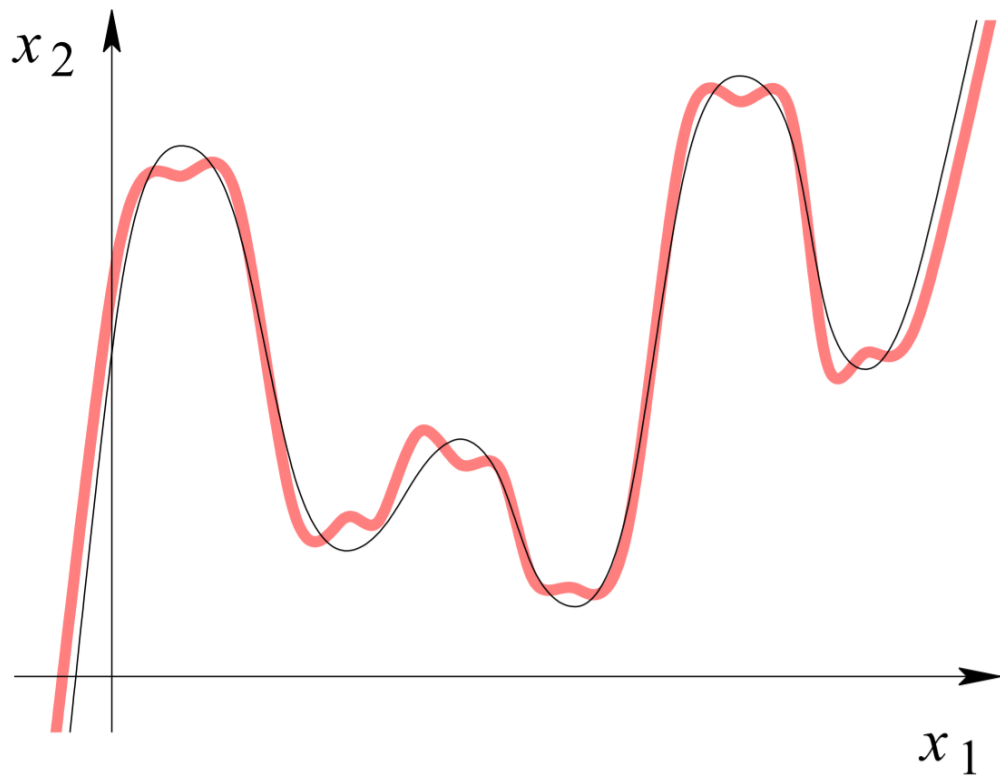
BOTTLENECK DISTANCE

[CohenSteiner-Edelsbrunner-Harer 2005]

$$d_B(D_g(f), D_g(g)) := \min_{\pi: \text{perfect matching}} \dots$$

$$\max \left(\begin{array}{l} \text{birth}(f_i) - \text{birth}(\pi(f_i)) \\ \text{death}(f_i) - \text{death}(\pi(f_i)) \end{array} \right)$$





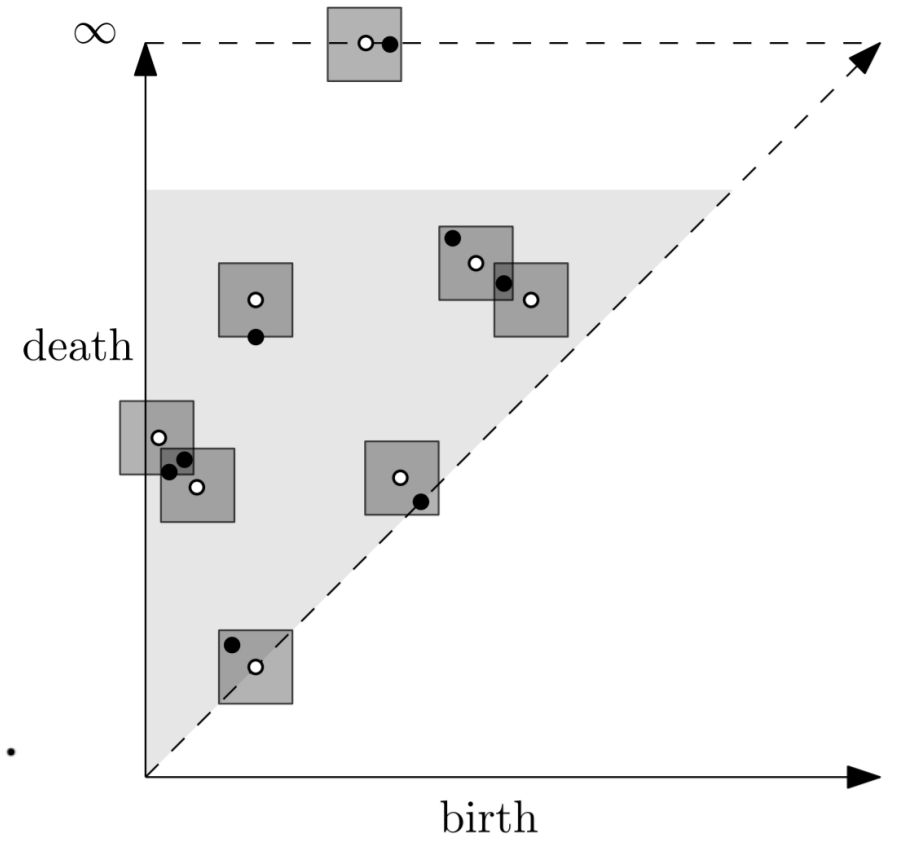
STABILITY THEOREM

[CohenSteiner-Edelsbrunner-Harer 2005] [CS-E-Morozov 2006]

$$d_B(D(f), D(g)) \leq \| f - g \|_\infty$$



PROOF OF STABILITY THEOREM.

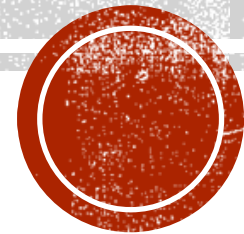


$$\begin{aligned}d_B(\mathbb{D}_p(\bar{f}_r), \mathbb{D}_p(\bar{f}_s)) &\leq \|\bar{f}_r - \bar{f}_s\|_\infty \\ &= (s - r) \cdot \|\bar{f} - \bar{g}\|_\infty.\end{aligned}$$

$$\begin{aligned}d_B(\mathbb{D}_p(\bar{f}), \mathbb{D}_p(\bar{g})) &\leq \sum_{i=0}^k d_B(\mathbb{D}_p(\bar{f}_{t_i}), \mathbb{D}_p(\bar{f}_{t_{i+1}})) \\ &\leq \|\bar{f} - \bar{g}\|_\infty \sum_{i=0}^k (t_{i+1} - t_i).\end{aligned}$$

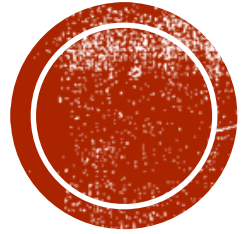


INTERMISSION



FOOD FOR THOUGHT.

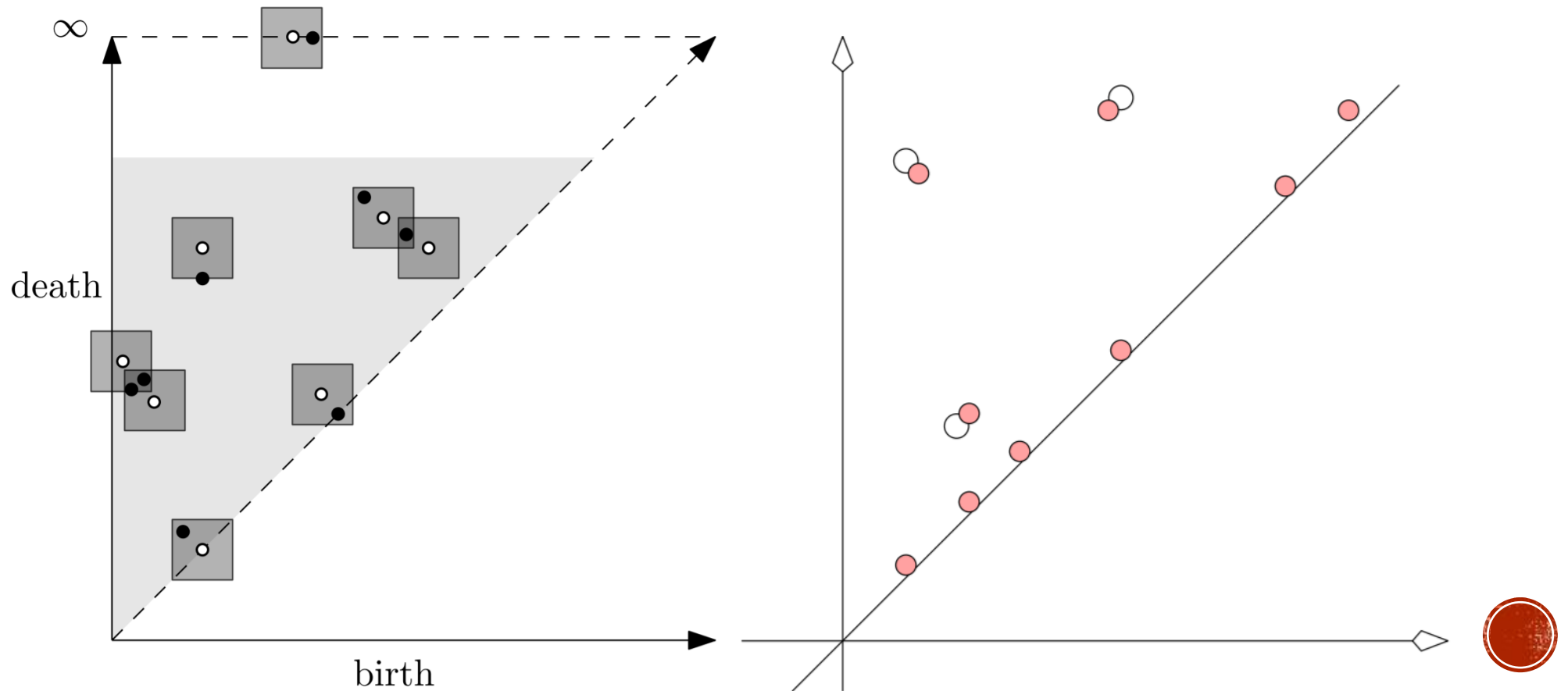
Pros and cons of using persistence diagrams?



COMPUTATION AND GENERALIZATION



COMPUTING PERFECT MATCHING



k-center

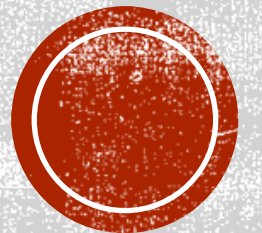
Given $\underline{D}_k \subseteq \underline{D}$, the multiplicities are defined by number of rev. NN's

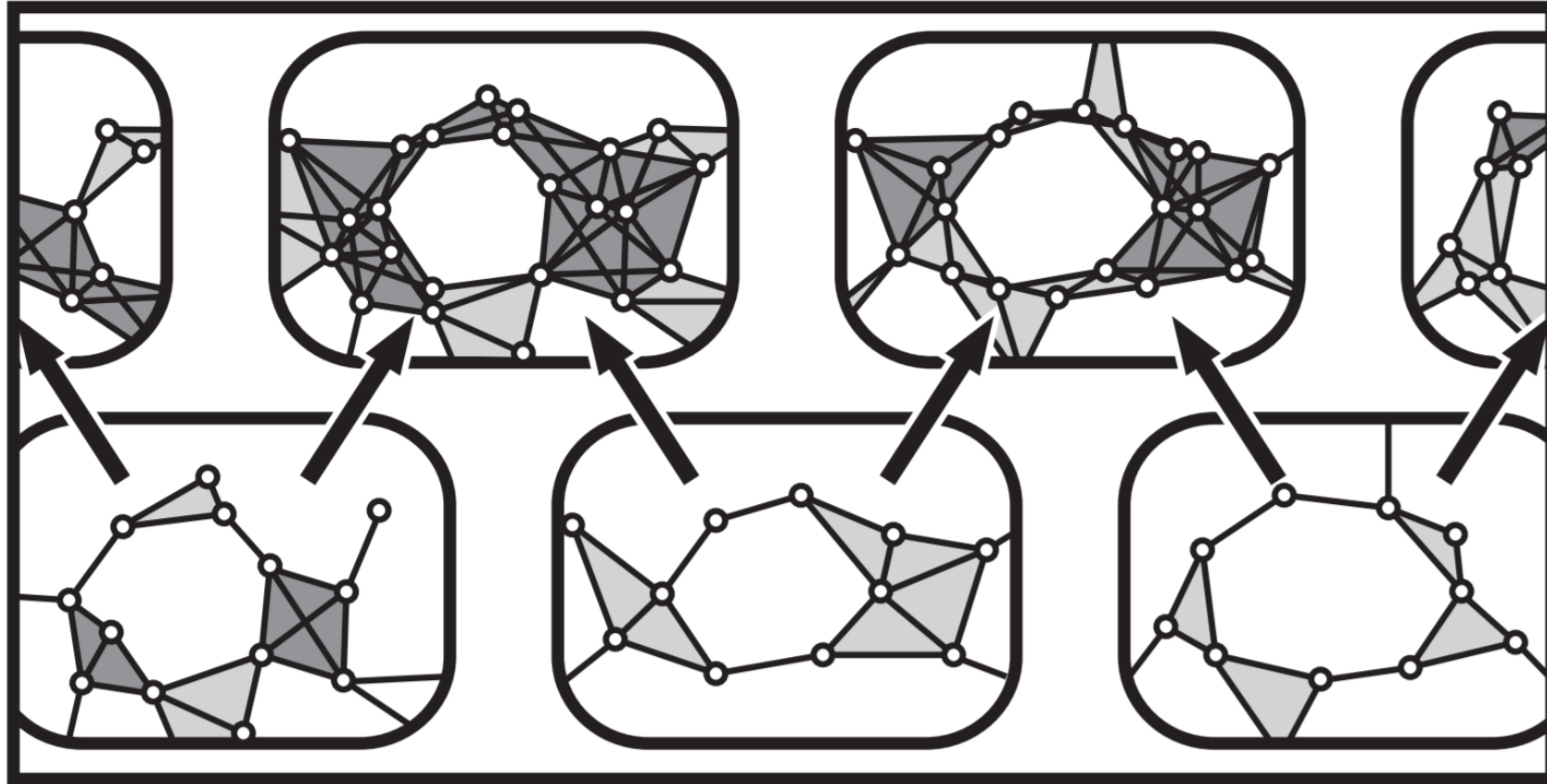
$d_B(\underline{D}, \underline{D}_k) = d_H(\underline{D}, \underline{D}_k)$

SKETCHING PERSISTENCE DIAGRAMS

[Sheehy-Sheth 2020]

Computing sequence of diagrams of k points converging to optimal, each a 2-approximation to OPT_k

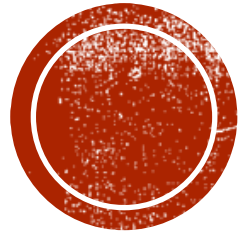




ZIGZAG

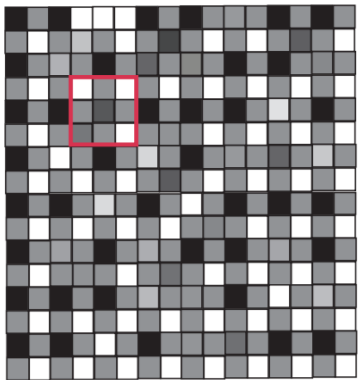
PERSISTENCE





APPLICATIONS



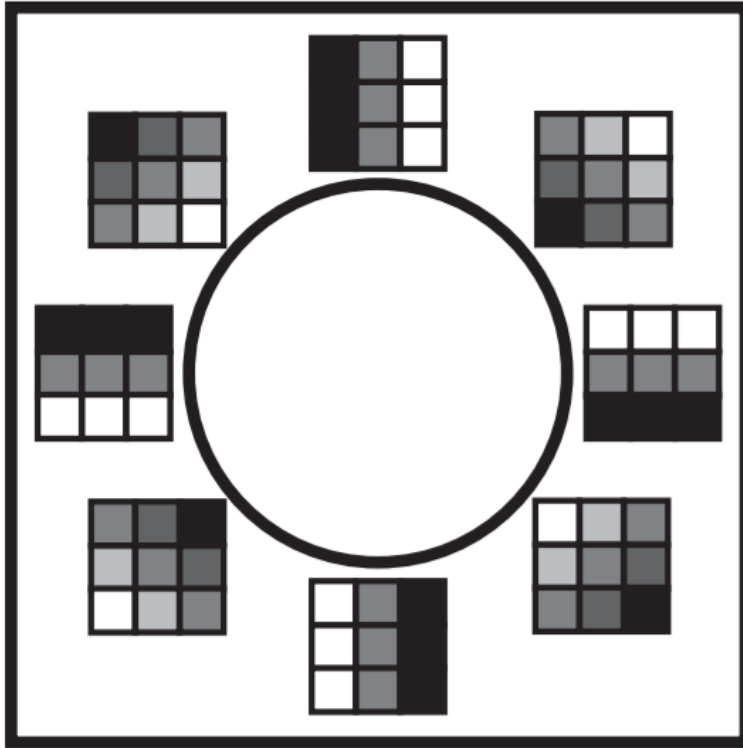
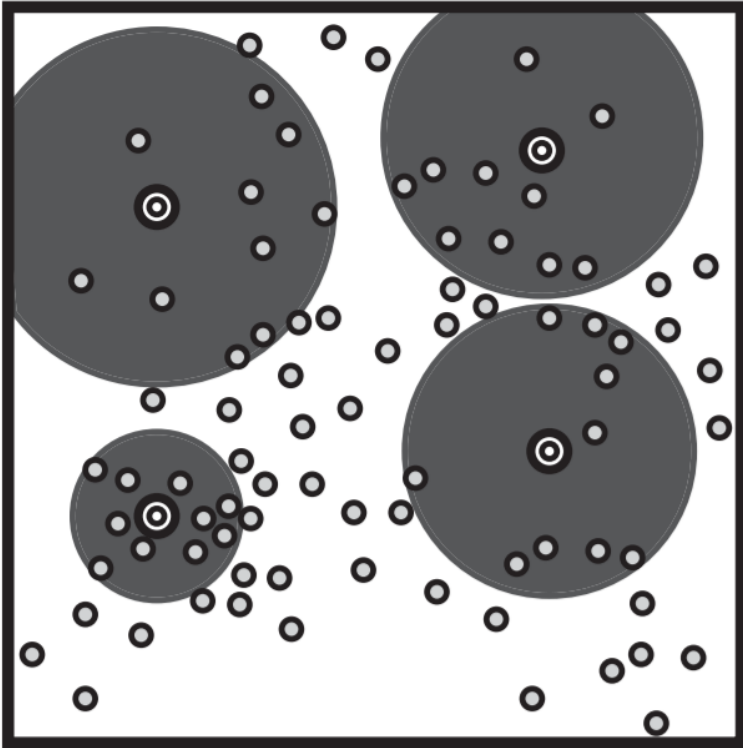


NATURAL IMAGES

[Lee-Mumford-Pedersen 2003]
[Carlsson-Ishkhanov-DeSilva-Zomorodian 2008]

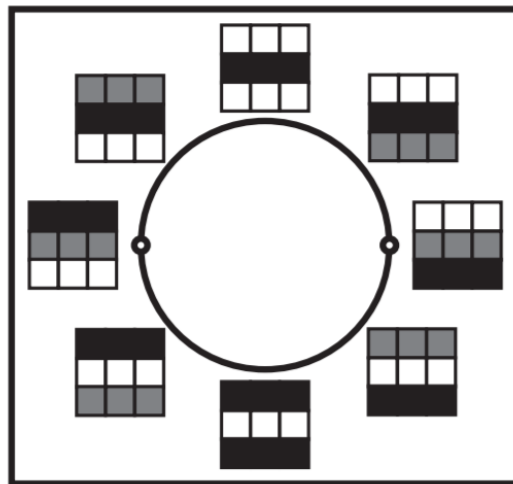
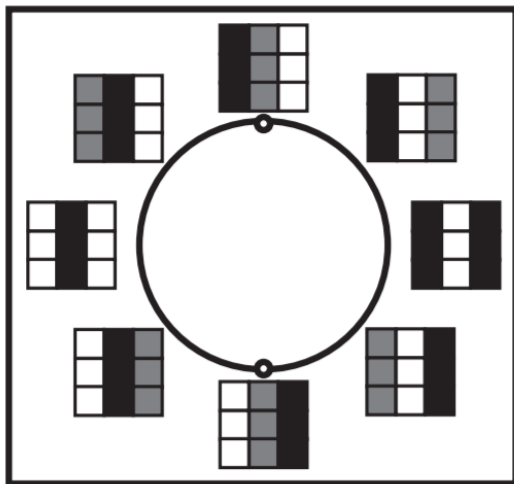
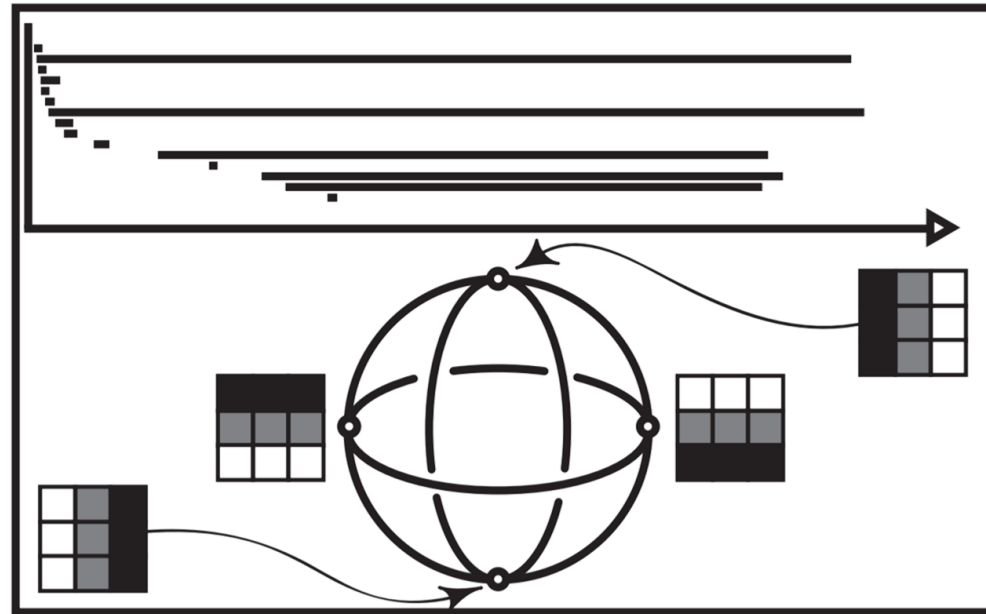
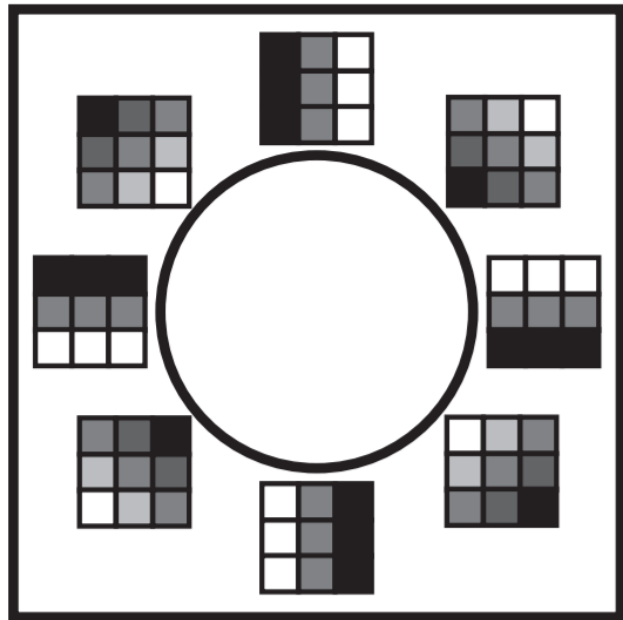


NATURAL IMAGES



[Lee-Mumford-Pedersen 2003]
[Carlsson-Ishkhanov-DeSilva-Zomorodian 2008]



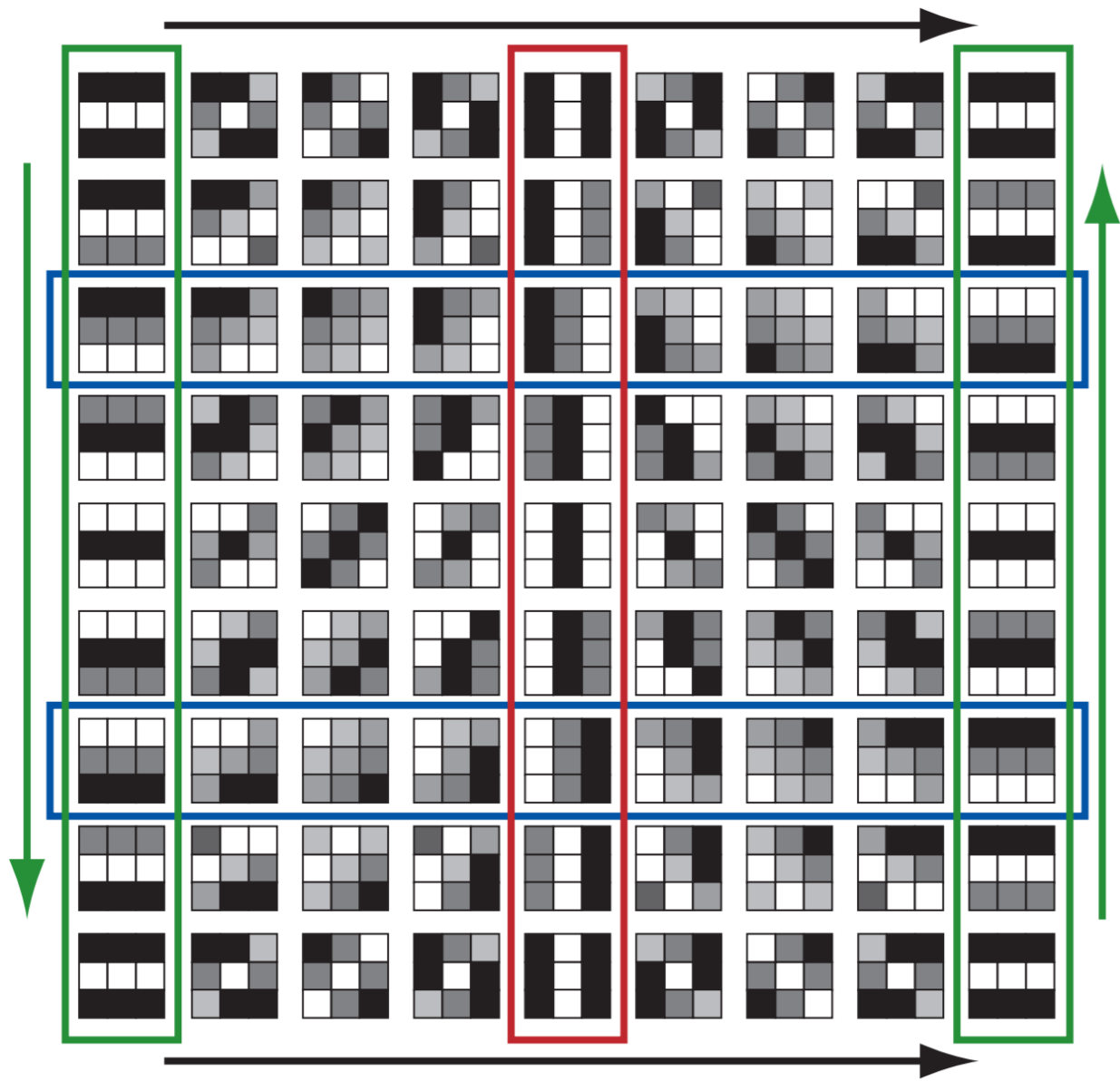


NATURAL IMAGES

[Lee-Mumford-Pedersen 2003]

[Carlsson-Ishkhanov-DeSilva-Zomorodian 2008]





NATURAL IMAGES

[Lee-Mumford-Pedersen 2003]
[Carlsson-Ishkhanov-DeSilva-Zomorodian 2008]



(d) -3

3	2	1	1	2	1	1	1	-1	1	2	3	4
2	1	-1	-1	1	-1	-1	-1	-1	1	2	3	4
1	-1	-2	-2	-1	-2	-2	-2	-2	-1	1	2	3
1	-1	-2	-2	-1	-2	-3	-2	-1	1	2	3	4
2	1	-1	-1	1	-1	-2	-1	1	2	3	4	5
3	2	1	1	2	1	-1	1	2	3	3	3	4
4	3	2	2	3	2	1	2	3	3	2	2	3
5	4	3	3	4	3	2	3	3	2	1	1	2
6	5	4	4	5	4	3	3	2	1	-1	-1	1
7	6	5	5	5	4	3	2	1	-1	-2	-1	1
8	7	6	6	6	5	4	3	2	1	-1	1	2

-2

3	2	1	1	2	1	1	1	-1	1	2	3	4
2	1	-1	-1	1	-1	-1	-1	-1	1	2	3	4
1	-1	-2	-2	-1	-2	-2	-2	-2	-1	1	2	3
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6	5	4	4	5	4	3	3	2	1	-1	-1	1
7	6	5	5	5	4	3	2	1	-1	-2	-1	1
8	7	6	6	6	5	4	3	2	1	-1	1	2

-1

3	2	1	1	2	1	1	1	-1	1	2	3	4
2	1	-1	-1	1	-1	-1	-1	-1	1	2	3	4
1	-1	-2	-2	-1	-2	-2	-2	-2	-1	1	2	3
1	-1	-2	-2	-1	-2	-3	-2	-1	1	2	3	4
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6	5	4	4	5	4	3	3	2	1	-1	-1	1
7	6	5	5	5	4	3	2	1	-1	-2	-1	1
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1

3	2	1	1	2	1	1	1	-1	1	2	3	4
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6	5	4	4	5	4	3	3	2	1	-1	-1	1
7	6	5	5	5	4	3	2	1	-1	-2	-1	1
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2

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6	5	4	4	5	4	3	3	2	1	-1	-1	1
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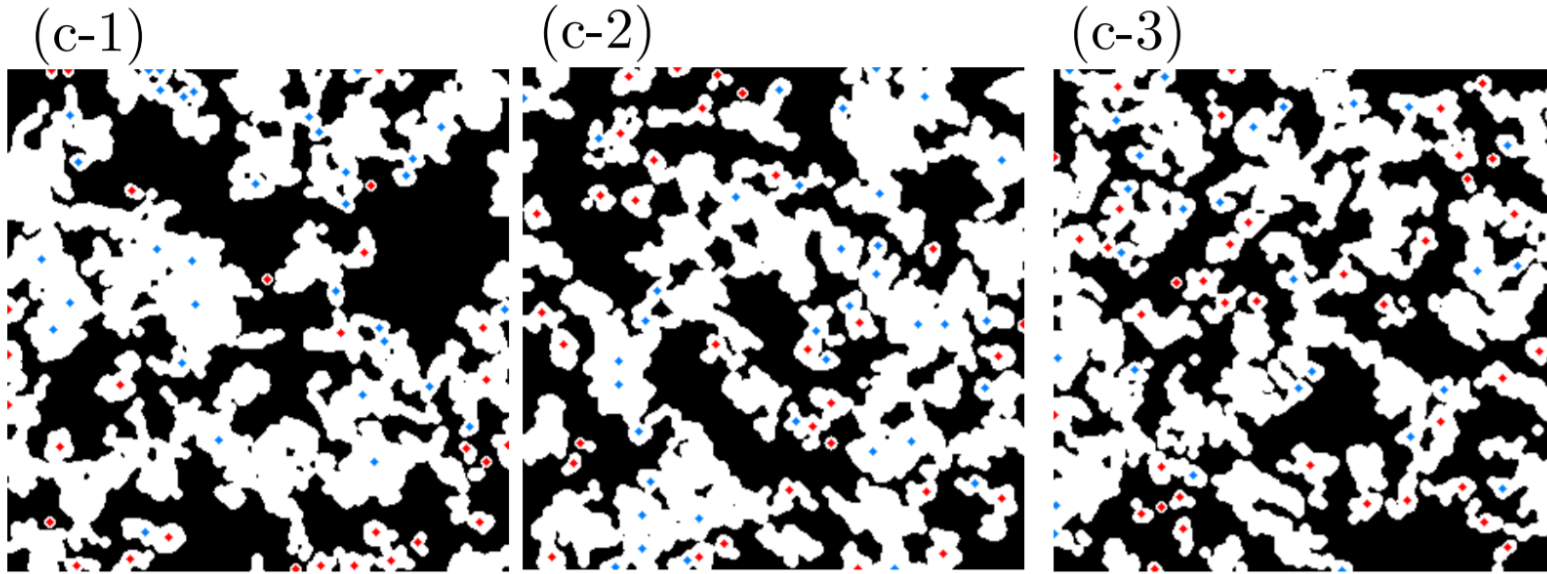
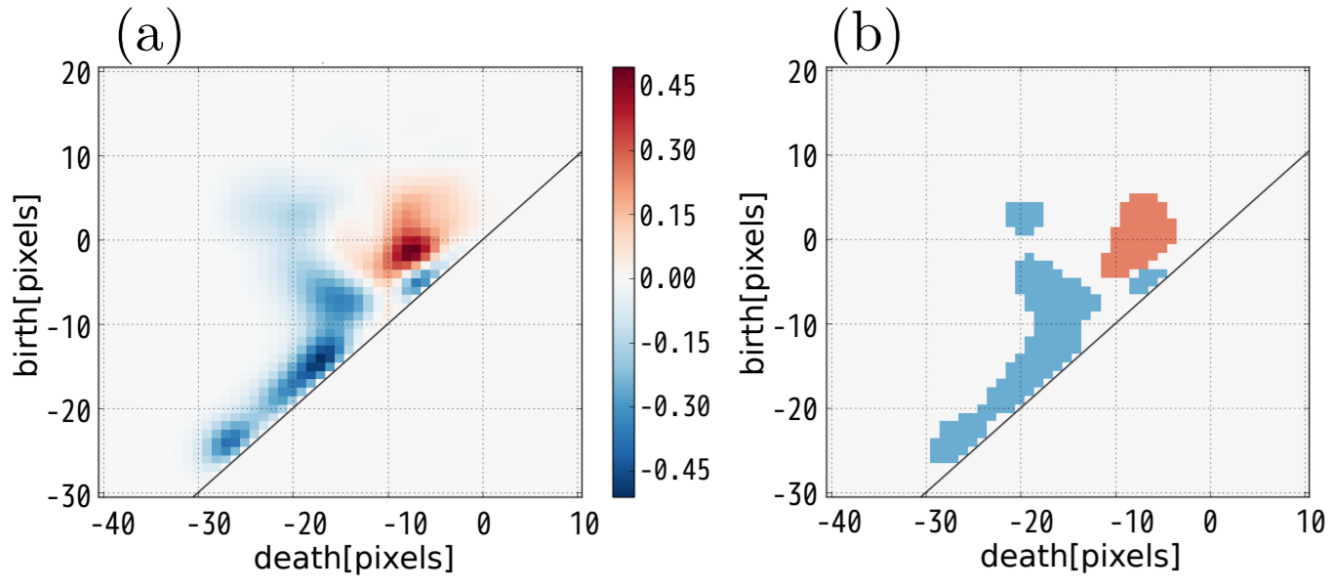
3

3	2	1	1	2	1	1	1	-1	1	2	3	4
2	1	-1	-1	1	-1	-1	-1	-1	1	2	3	4
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7	6	5	5	5	4	3	2	1	-1	-2	-1	1
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PERSISTENCE IMAGES



PERSISTENCE IMAGES



data

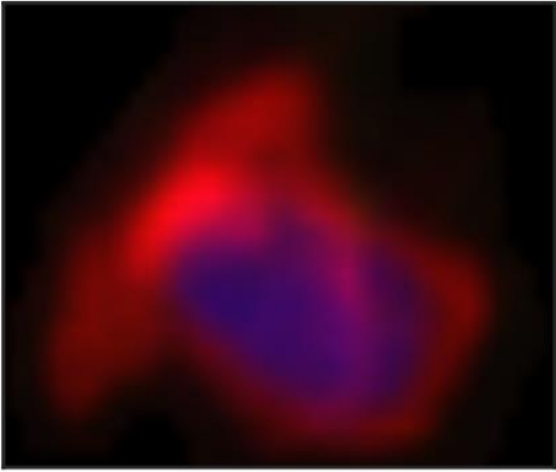


diagram B

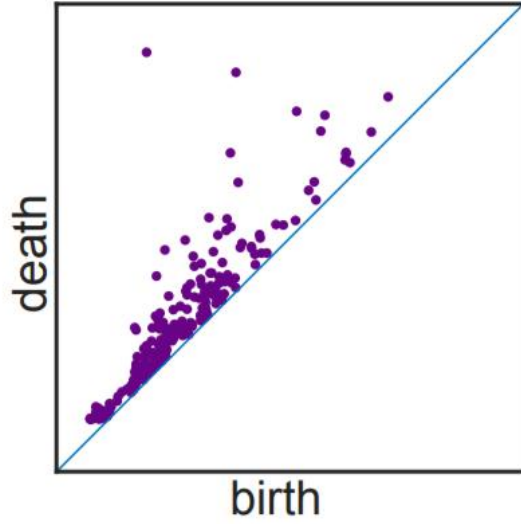
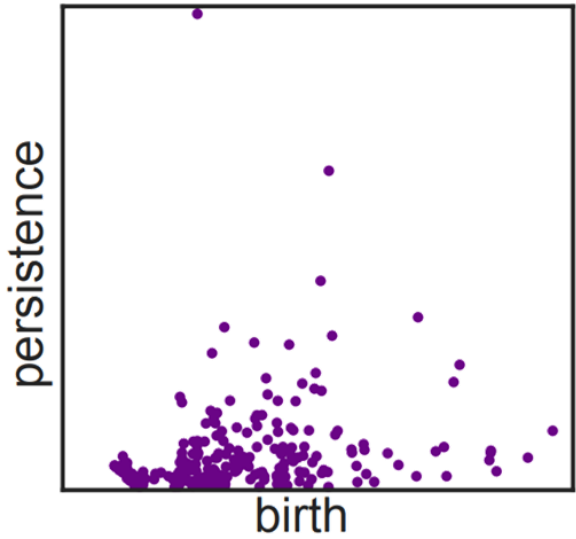
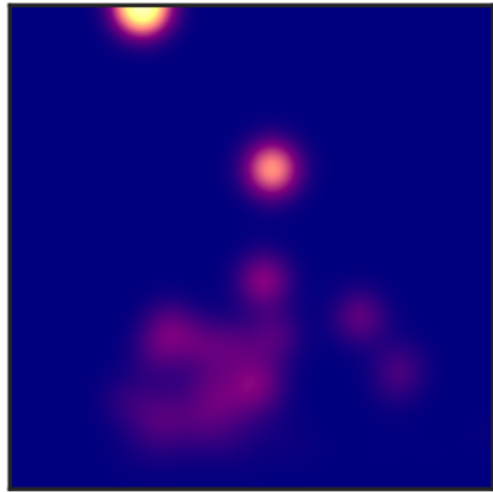


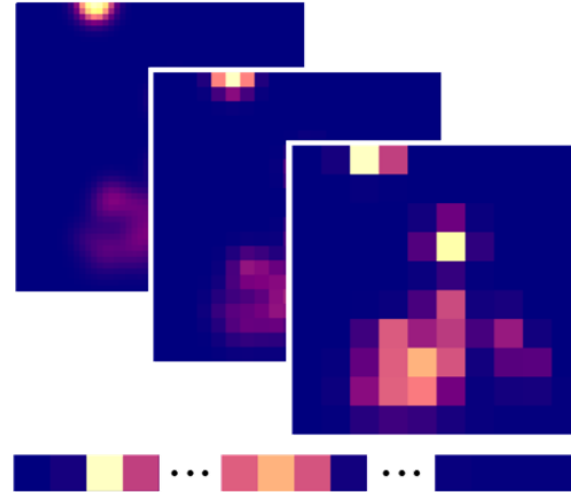
diagram $T(B)$



surface



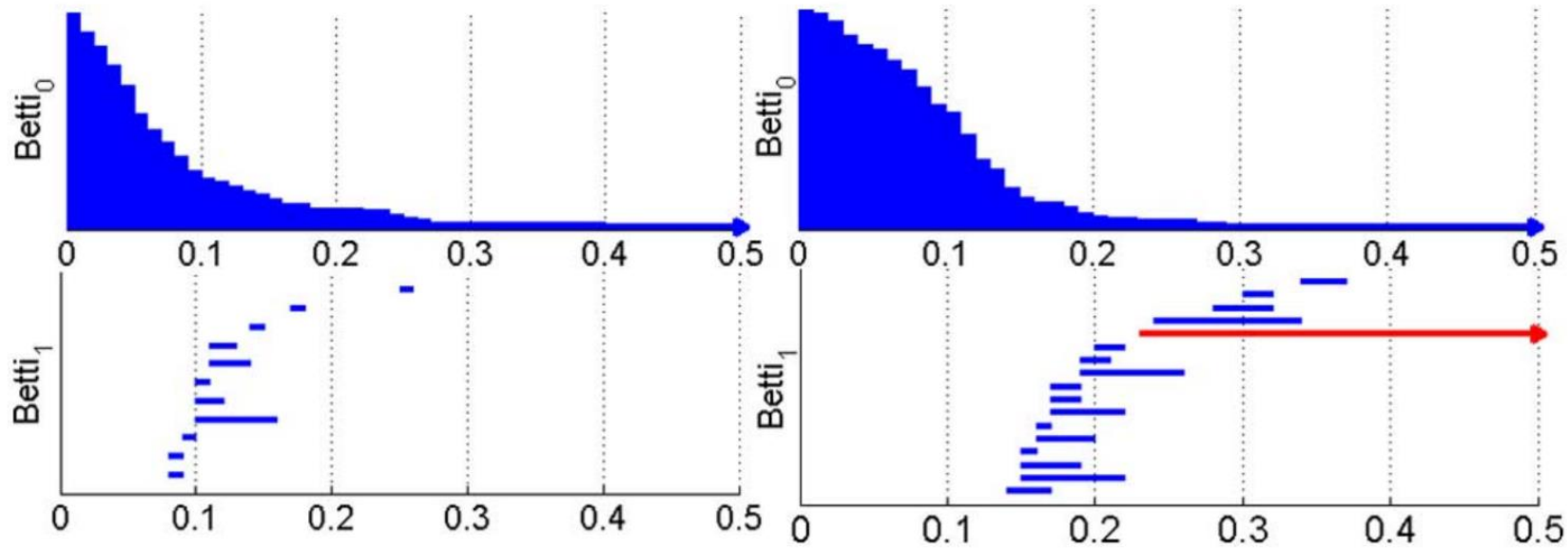
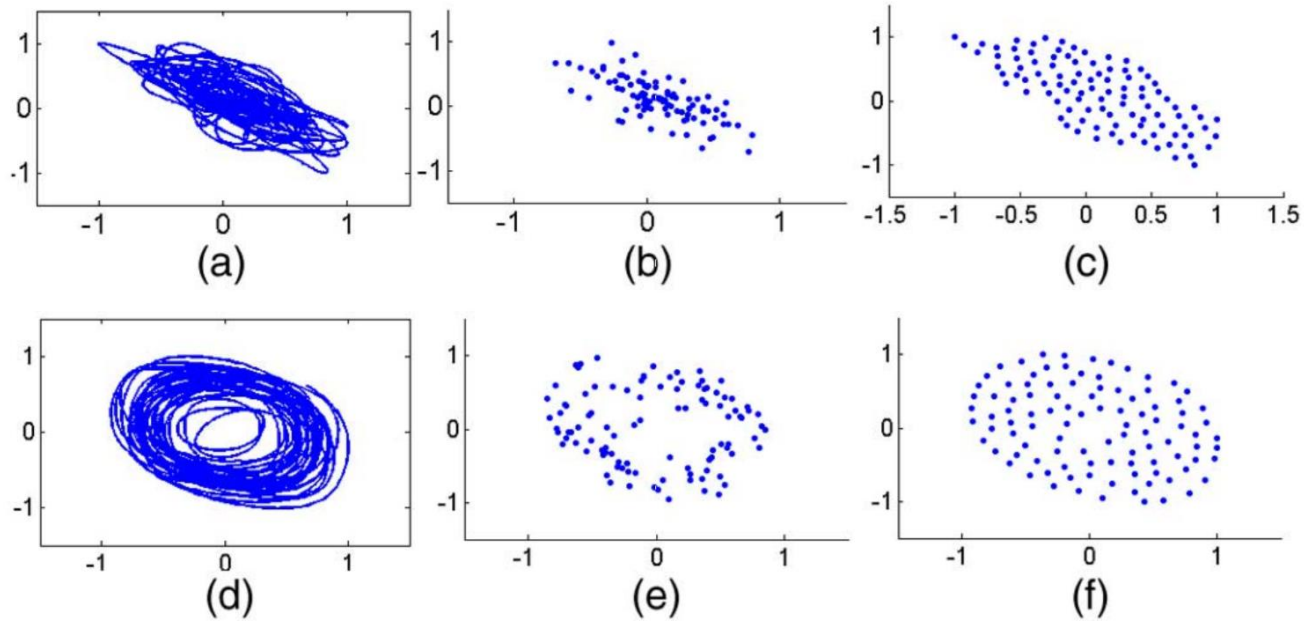
image



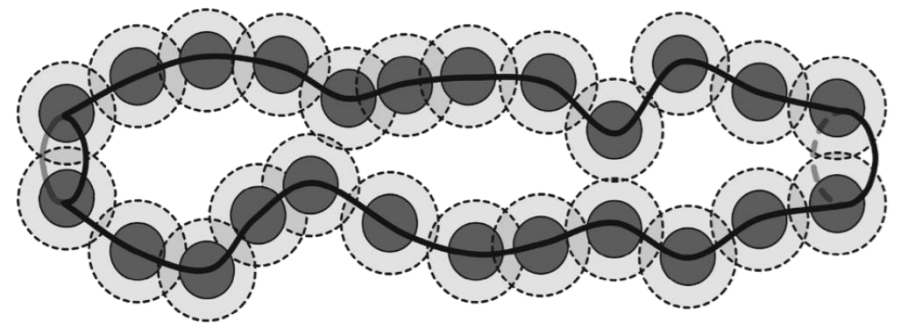
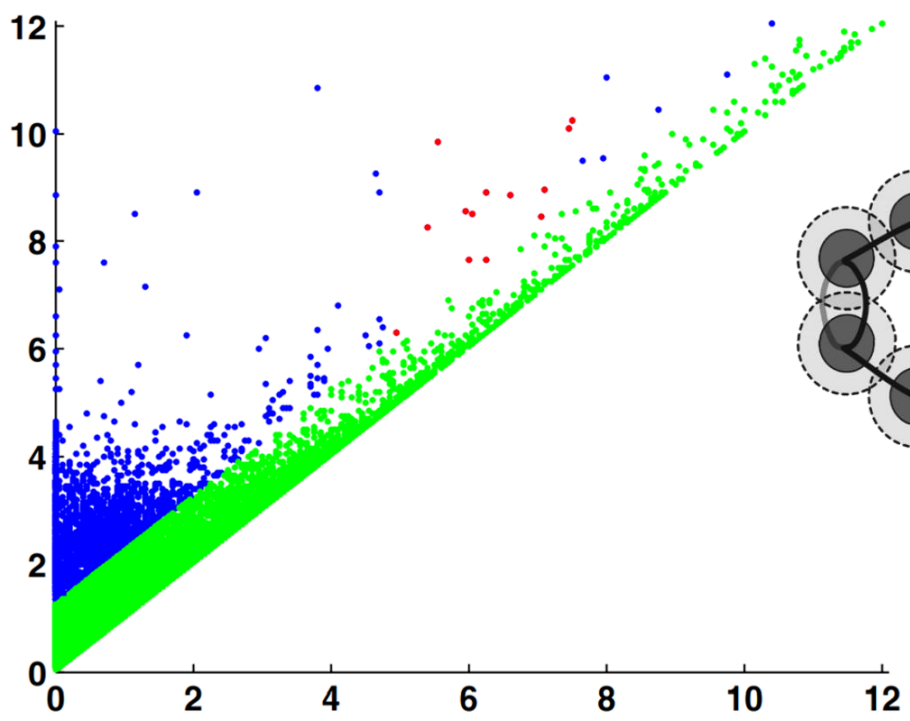
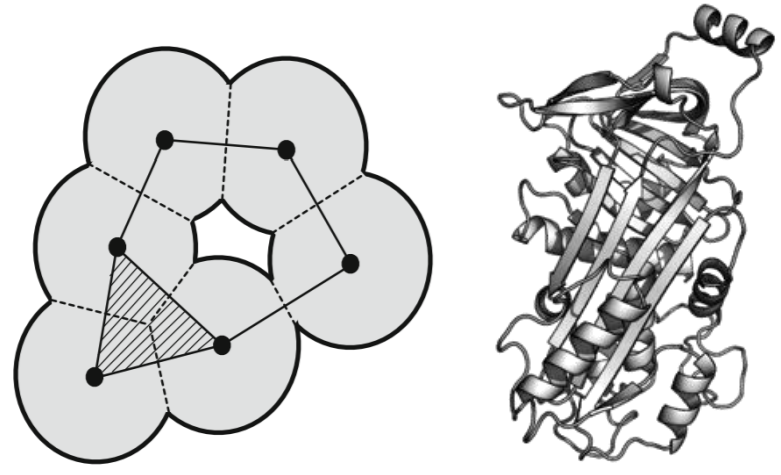
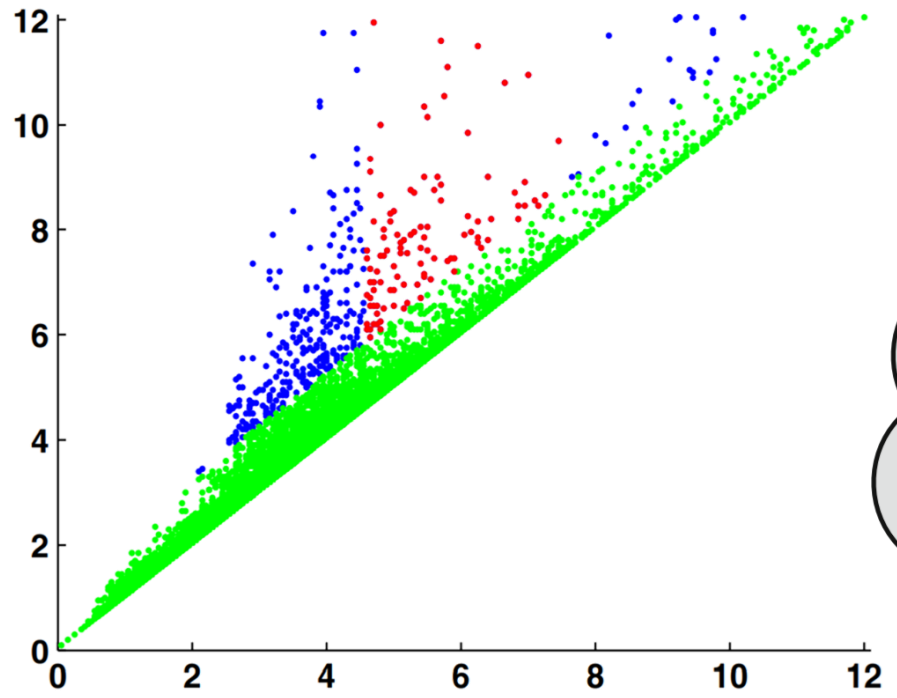
PERSISTENCE IMAGES



WHEEZE DETECTION



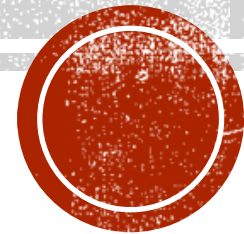
PROTEIN COMPRESSIBILITY



[Gameiro-Hiraoka-Izumi-
Kramar-Mischaikow-Nanda 2015]



SUMMARIZING DATA WITH A SINGLE IMAGE



NEXT TIME.

Water-rising is a cool idea.