1. ***Algorithmic engineering.*** In this question, you are allowed to use the following tools we build during the class as *black boxes*:

   - *multiple-source shortest paths* by Klein [4],
   - *cycle separators* and *r-division* by Frederickson [2] and Klein-Mozes-Sommer [5],
   - *Monge heaps* and *FR-Dijkstra algorithm* by Fakcharoenphol-Rao [1],

   or any of their precedents. All other tools on planar graphs (even if you find them in the literature) have to be built from scratch. In particular, you are *not* allowed to use

   - *linear-time shortest-path algorithm* by Henzinger-Klein-Rao-Subramanian [3]

   as a black box. (But if you can rebuild their algorithm from scratch ...)

   (a) Let G be an n-vertex undirected planar graph with *non-negative* edge weights. Design and analyze an algorithm that computes a single-source shortest path tree in $O(n \log\log n)$ time.
   [Hint: Dijkstra algorithm runs in $O(n \log n)$ time on planar graphs out-of-the-box.]

   (b) Let G be an n-vertex undirected planar graph, possibly with *negative* edge weights. Design and analyze an algorithm that computes a single-source shortest path tree in $O(n \operatorname{poly}\log n)$ time, or correctly reports that there is a negative cycle in G. What is the best running time you can get?

   ★ (c) The difficulty in improving the running time in (b) seems to be that computing MSSP takes $\Theta(n \log n)$ time in general, and the recursion takes $O(\log n)$ levels. Can you improve the running time?
   [Hint: If you solve this problem, even with a $\log^* n$-factor over the state-of-art, you can publish a paper.]

References

[1] Jittat Fakcharoenphol and Satish Rao. Planar graphs, negative weight edges, shortest paths, and near linear time. *Journal of Computer and System Sciences* 72(5):868–889, 2006.

[2] Greg N. Frederickson. Fast Algorithms for Shortest Paths in Planar Graphs, with Applications. *SIAM Journal on Computing* 16(6):1004–1022, 1987.

[3] Monika R. Henzinger, Philip Klein, Satish Rao, and Sairam Subramanian. Faster shortest-path algorithms for planar graphs. *J. Comput. Syst. Sci.* 55(1):3–23, 1997. ⟨http://dx.doi.org/10.1006/jcss.1997.1493⟩.

[4] Philip N Klein. Multiple-source shortest paths in planar graphs. *Proceedings of the sixteenth annual ACM-SIAM symposium on Discrete algorithms*, 146–155, 2005.

[5] Philip N. Klein, Shay Mozes, and Christian Sommer. Structured recursive separator decompositions for planar graphs in linear time. *Proceedings of the 45th annual ACM symposium on Symposium on theory of computing - STOC '13*, p. 505, 2013. ACM Press.