1. ***Folding polygons.*** We showed in class that the Latin cross can be folded into both a cube and a tetrahedron. A natural restriction of polygon folding is to require that each edge has to be glue to another whole edge of the polygon. We call this an ***edge-to-edge gluing***.

   (a) How many combinatorially different polyhedra can be folded from the Latin cross using edge-to-edge gluings? Prove your claim.

   If ***arbitrary gluings*** are allowed (where vertices of the polygon can be glued to the interior of an edge), we have more possibilities. In fact, there are 23 different (non-congruent) convex polyhedra the Latin cross can fold into.

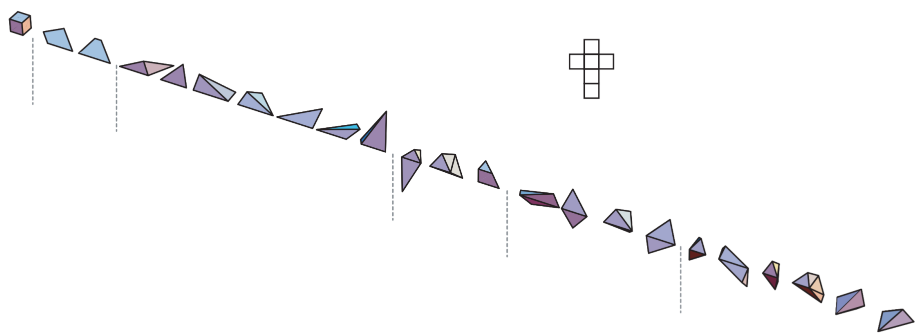

Figure 1. The 23 non-congruent convex polyhedra folded from the Latin cross. (From *Geometric Folding Algorithms* by Demaine and O'Rourke, Figure 25.30.)

   (b) Prove that any *convex* polygon folds into *infinitely* many different convex polyhedra.

   (c) What is the maximum number of vertices you can have among all convex polyhedra folded from a convex k-gon?

   Latin cross is an example of *non-convex* polygon. Unlike convex polygons, some non-convex polygon only folds into finitely many different polyhedra (under arbitrary gluings).

   (d) Construct a non-convex polygon that cannot be folded into any convex polyhedron.

   (e) Construct a non-convex polygon that folds into infinitely many convex polyhedra.

   ★(f) Given a non-convex polygon P that can be folded into only *finitely* many convex polyhedra, where all edges of the polygon has integer length.

   Prove that every the convex polyhedra foldable from P using arbitrary gluing can in fact be achieved by first subdividing every edge of P right at the middle point, then performing an edge-to-edge gluing of the subdividing polygon.

   ★(g) Does every (not necessarily convex) polygon fold into some (possibly non-convex) polyhedron?