1. ***Generating mazes.***

   Consider a rectangular grid. For each square in the grid, add either a *diagonal* (⟍) or an *anti-diagonal* (⟋). The diagonals connect into paths on the rectangular grid (see Figure 1 for an illustration).

   Prove that there is a path either from the top to the bottom, or from the left to the right of the rectangle, using only the diagonals and the anti-diagonals.

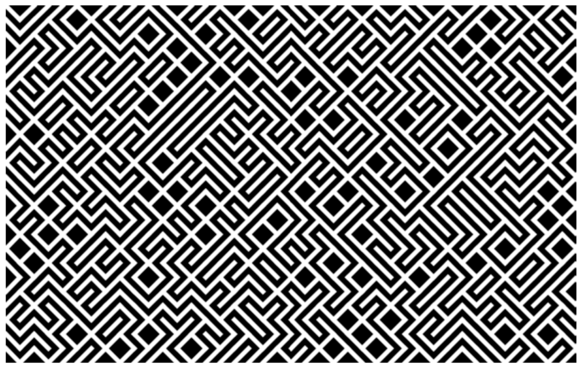

Figure 1. A random maze generated from random diagonals. From *10 PRINT: CHR$(205.5+RND(1)); : GOTO 10* by Montfort *et al.*, November 2012. Shared under a Creative Commons BY-NC-SA 3.0 license.

2. ***Proving Jordan curve theorem.*** Remember our old friend — the *Jordan curve theorem* — from the first lecture? Both of you have grown so much since; last time you met her she was just a little polygon theorem, and you were new to the whole topology business.

   (a) Read about the *Mayer-Vietoris sequence* from any source you like. State a version of the Mayer-Vietoris sequence here, in your own words.

   (b) Prove that for any function $f : [0,1]^k \to S^n$, one has $H_i(S^n - f([0,1]^k)) \cong 0$ for any $i > 0$ and $H_0(S^n - f([0,1]^k)) \cong \mathbb{Z}$, using Mayer-Vietoris sequence.

   [Hint: If there is a nontrivial cycle α in $S^n - f([0,1]^k)$, argue that α remains nontrivial in either

   $$S^n - f([0,1]^{k-1} \times [0,0.5]) \quad \text{or} \quad S^n - f([0,1]^{k-1} \times [0.5,1]).$$

   Repeat the argument ad infinitum one produces a sequence of nested intervals $I_1 \supset I_2 \supset \cdots$ where α is nontrivial in any of the $S^n - f([0,1]^{k-1} \times I_j)$. Derive a contradiction.]

   (c) Prove the Jordan curve theorem.

   [Hint: Remember, the Jordan curve theorem is a statement about the complement of the curve.]

   (You are allowed to use any resources available; so this is really a test of literacy, seeing if you can read and communicate your ideas using the language of homology. Find the right tools, state them correctly, and prove that the spaces you apply on satisfy all the requirements of the statement. You may use any standard results from algebraic topology without proofs.)