1. ***Untangling mess.*** A ***drawing*** of a graph G is a mapping from G to the plane, where each vertex maps to a point in $\mathbb{R}^2$ and each edge maps to a continuous curve between the two points that correspond to the two endpoints of the edge. For this problem, it is safe to assume that there are only finite many intersections in the drawing of G.

   Graph G is ***planar*** if G has a drawing in the plane such that no edge passes through a vertex and no pair of edges intersect (except at the endpoints); in other words, graph G can be *embedded* into the plane.

   (a) Prove that if the input graph G has a drawing where every pair of edges intersect an even number of times, then G is a planar graph. *[Hint: You can start by assuming that G is a cycle (of arbitrary length), which is worth partial credit.]*

   A pair of edges is ***independent*** if the two edges do not share any common endpoints.

   ★(b) Prove that if the input graph G has a drawing where every pair of *independent* edges are disjoint, then G is a planar graph. (There are no restrictions on the number of intersections between a pair of edges that share a common endpoint.)

2. ***Regular homotopy of polygons.*** Recall that during the class we defined regular homotopy as an arbitrary sequence of 0→2, 2→0, and 3→3 moves, when the input curves are assumed to be *generic*.

   (a) Provide a set of operations that is equivalent to regular homotopy for *closed polygonal curves*, and explain why your definition is the proper analog to the generic setting. (A regular homotopy for polygons has to always turn a polygon into another polygon.)

   (b) How many moves do you need to turn a polygon with rotation number 1 into a simple polygon, using the set of operations defined in (a)? Can you beat quadratic?