1. ***Generating mazes.***

   Consider a rectangular grid. For each square in the grid, add either a *diagonal* (⟍) or an *anti-diagonal* (⟋). The diagonals connect into paths on the rectangular grid (see Figure 1 for an illustration).

   Prove that there is a path either from the top to the bottom, or from the left to the right of the rectangle, using only the diagonals and the anti-diagonals.

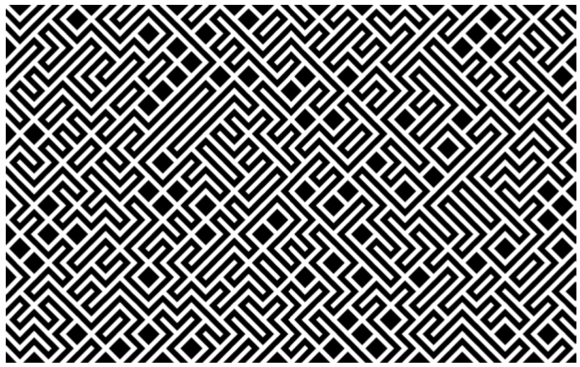

Figure 1. A random maze generated from random diagonals. From *10 PRINT: CHR$(205.5+RND(1)); : GOTO 10* by Montfort *et al.*, November 2012. Shared under a Creative Commons BY-NC-SA 3.0 license.

2. ***Proving Jordan curve theorem.*** Remember our old friend — the *Jordan curve theorem* — from the first lecture? Both of you have grown so much since; last time you met her she was just a little polygon theorem, and you were new to the whole topology business.

   Prove the Jordan curve theorem using the tool from homology. You may use any standard results from algebraic topology without proving them. For full credit, find the right tools, state them correctly, and prove that the objects you apply on satisfy all the requirements of the statement. Extra credit if the proof involves analyzing long exact sequence.

   [Hint: Remember, the Jordan curve theorem is a statement about the complement of the curve. Also, you are allowed to use any resources available; so this is really a test of literacy, seeing if you can read and communicate your ideas using the language of homology.]

3. ***Shortest-path complex.*** Let G be an undirected graph with non-negative edge weights. For convenience, we assume G to be connected as well. Consider the collection of all *shortest paths* in graph G; without loss of generality, we can assume that there is a *unique* shortest path between any two vertices in G by perturbing the edge weights a little. We built the ***shortest-path complex*** of G, denoted as $\boldsymbol{\Pi_G}$, by treating each shortest path in G as a node (0-simplex) of Π_G and create a k-simplex on $k+1$ nodes in Π_G if the $k+1$ corresponding shortest paths have a common intersection.

   (a) Prove that the shortest-path complex Π_G is homotopically equivalent to the graph G.

   (b) Prove that if G is a tree, then the collection of pairwise intersecting shortest paths must intersect at a common vertex.