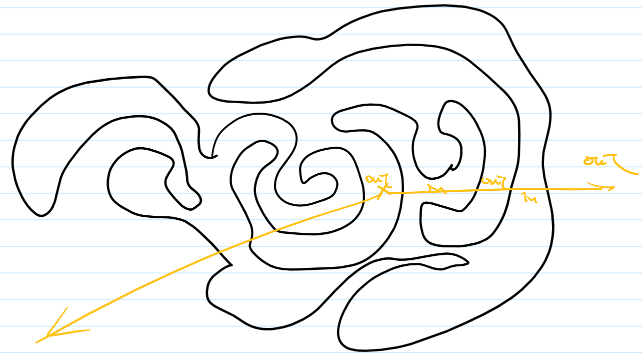
Administrivia.

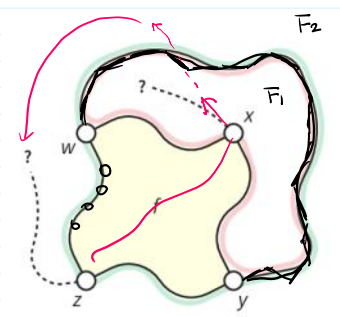
- HW 1. due. HW 2 out soon.
- Contact me if your presentation date is close!

Planar graphs have ... Schnyder Woods



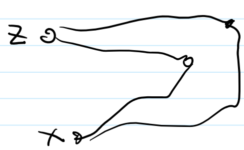
Setup: Plane triangulation

Motivation: 1. Peeling off Δs one-by-one, preserving $\chi = 1$.
2. Nash-Williams tree-decomposition
3. Straight-line embedding.

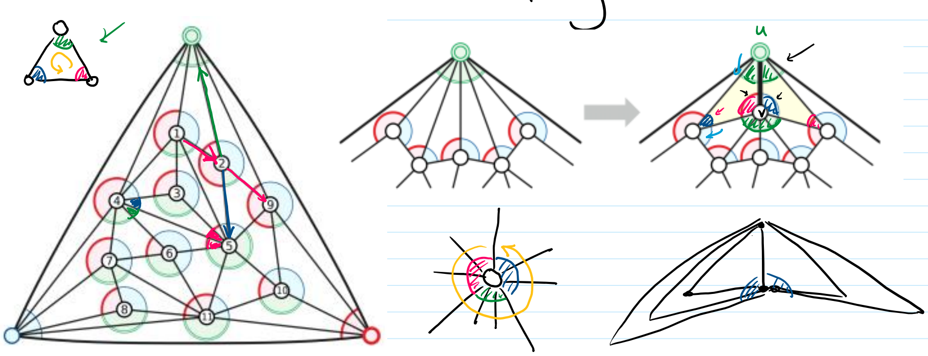


Claim. All simple planar graphs can be Δlated.

pf. Let f be a face w/ deg ≥ 4



Idea: Tri-color corners consistently.



Claim. At least one edge uv has u & v share exactly two neighbors.

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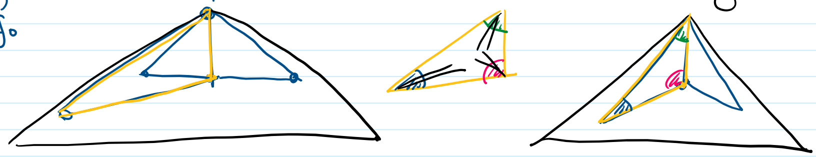
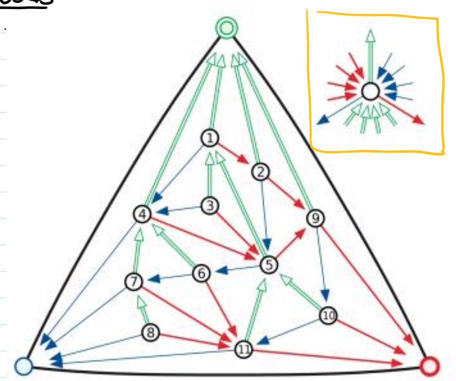
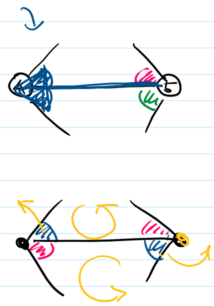
Pf.



[Schnyder '89]

Schnyder woods





- Another proof of Euler's formula!

#edges =: m #vertices =: n. #faces =: f. $n-m+f=2$

$$m = 3(n-3)+3 = 3n-6$$

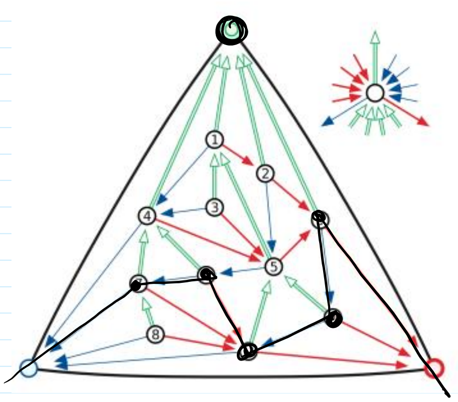
$$\#\text{corners} = 3f = \boxed{\sum_v \deg v = 2m = 6n-12} \Rightarrow f = 2n-4$$

$$n-m+f = \cancel{n} - (\cancel{3}n-6) + (\cancel{2}n-4) = 2 \;!$$ ☒

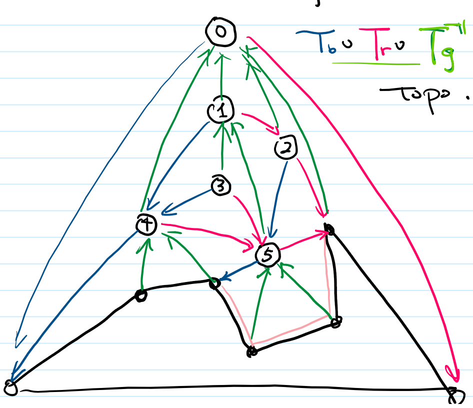
Canonical ordering / Shelling order

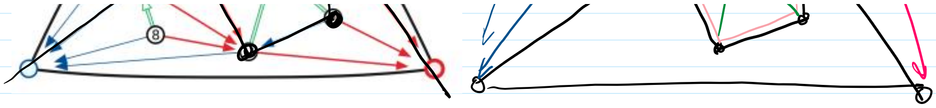
Total order of vertices such that:

1. $G[v_{k+1},\dots,v_n]$ 2-connected. contains base
2. v_k has neighbors on $G[v_{k+1},\dots,v_n]$ forming subint. of size ≥ 1



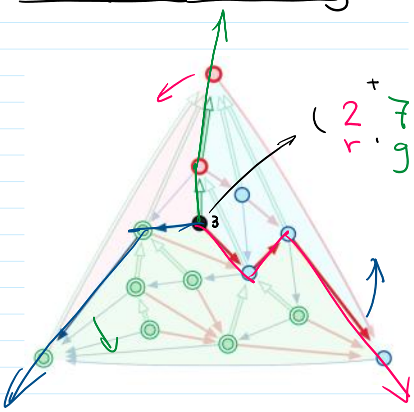
$\equiv$ $T_b \cup T_r \cup T_g$ dag. Topo. order.



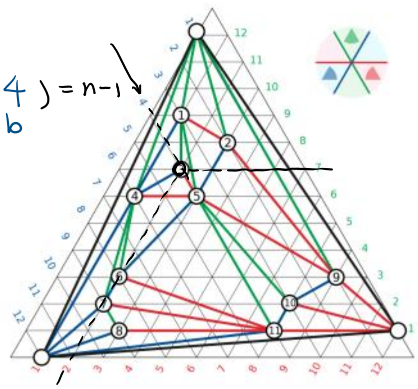


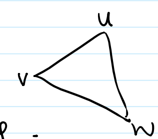
Straight-line embedding on grids

$x + y + z = n-1$



$(\overset{r}{2} + \overset{g}{7} + \overset{b}{4}) = n-1$





Claim. All $\triangle$s oriented ccw in Schnyder drawing.

pf.
$$\begin{vmatrix} 1 & g(u) & b(u) \\ 1 & g(v) & b(v) \\ 1 & g(w) & b(w) \end{vmatrix} = (g(v)-g(u))(b(w)-b(u)) - (g(w)-g(u))(b(v)-b(u))$$
$$> 0$$

— ◎ —

Planar graphs have … α-Orientations

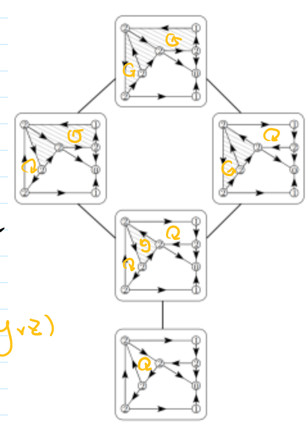
lattice structure?

Let $\alpha : V_G \to \mathbb{N}$.

An α-orientation is an edge orientation of G
s.t. $\text{out-deg}(v) = \alpha(v)$

[Ossona de Mendez '94] [Felsner '04], [Kauffmann '83] [Gilmer-Litherland '86]

Thm. Given any feasible α on plane graph G.
α-orientations of G form a distributive lattice
$(S, \leq, \vee, \wedge)$
$(x \wedge y) \vee z = (x \vee z) \wedge (y \vee z)$



Schnyder woods is a 3-orientation.

Markov chain generates uniform distri. over objects.

— ? —

Next time: Separators!

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