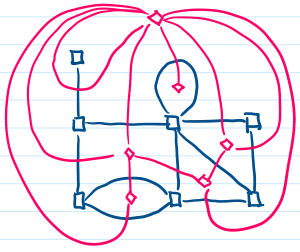
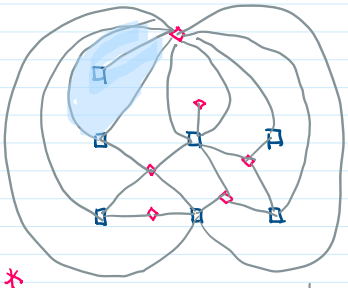
0408

Tuesday, September 24, 2024 10:07 AM

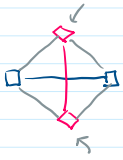
Planar graphs are ... Quad Meshs

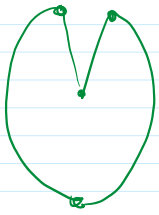


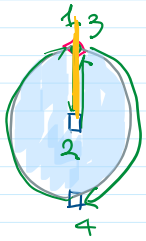
primal G & dual G^*



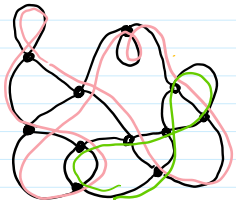
[Tait 1876-77. Steinitz 1916]

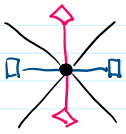




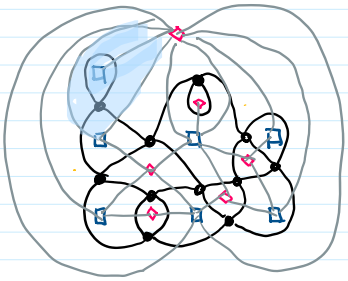


quad mesh
radial graph $G^\diamond$
(G^*)





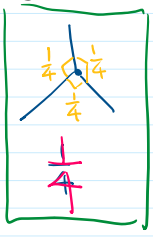
medial graph $G^\times$
$(G^*)^\times$



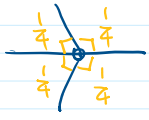
$(G^\times)^* = G^\diamond$

Prop. Any quad mesh of a sphere ($\chi = 2$) has a vertex of deg ≤ 3.

proof.



K $\frac{1}{4}$

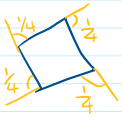


0

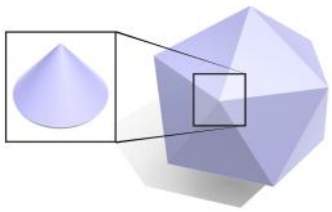
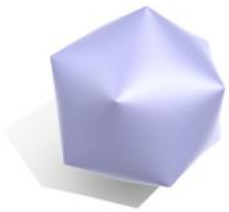
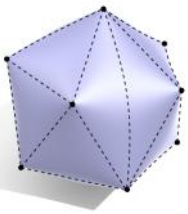
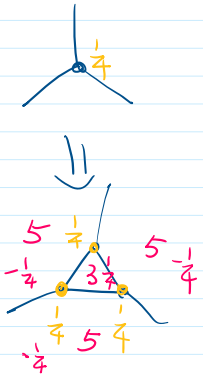
$-\frac{1}{4}$

$\sum_v K(v) = \chi = 1$

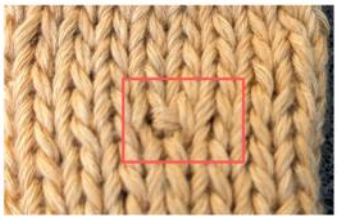
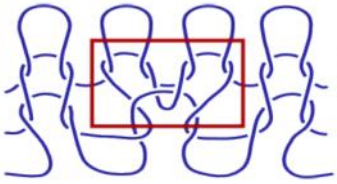
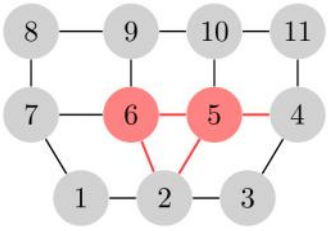
$+ \sum_f K(f) = 0$ (quads)

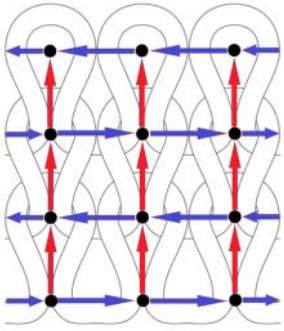


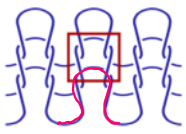
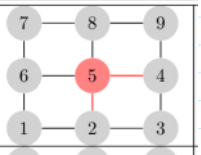
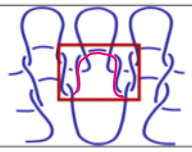
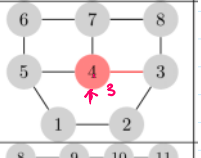
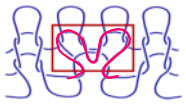
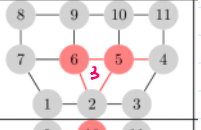
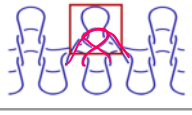
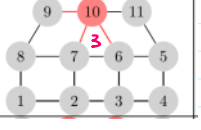
- Curvatures can be pushed & shifted around!

Example: Knitting.

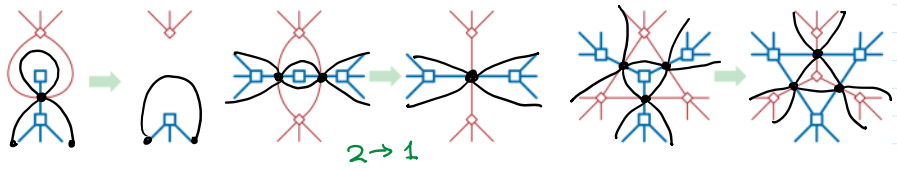


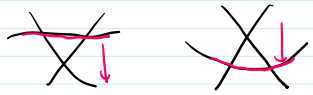
Stitch	In Fabric	Subgraph
Knit (k)		
Yarn over (yo)		
Knit Front and Back (kfb)		
Knit Two Together (k2tog)		

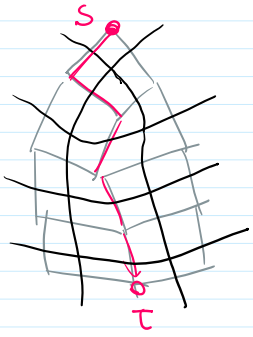
Planar graphs are ... Discrete, Metric

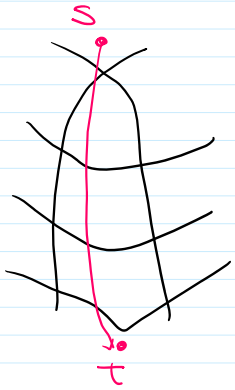
Planar graphs are ... Discrete Metric

Electrical Reduction.



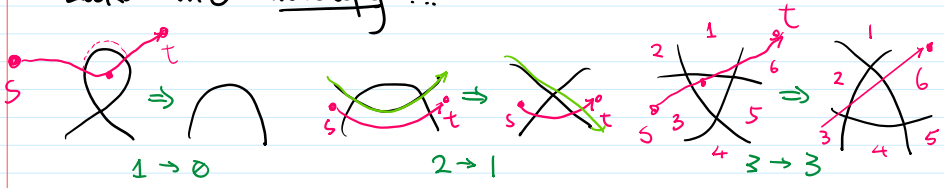






- shortest path. [Ahrens Jr. 1960].
- maximum flow

Looks like homotopy ...



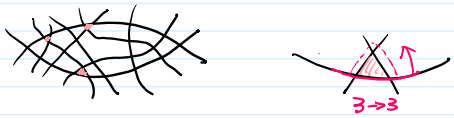
Any multicurve can be untangled using homotopy.
[Steinitz 1916, 34] [Fáry-Braun 1948] [Thrumper 1989] [de Graaf-Schrijver 1994] [CCE'09]

Thm. Any multicurve can be untangled using electrical moves.

Cor. Any plane graph is electrically reducible.

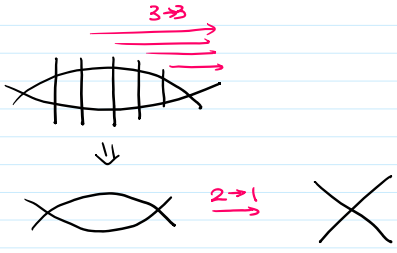
Proof. Bigon Reduction

1. find "minimal" bigon
   area-minimal

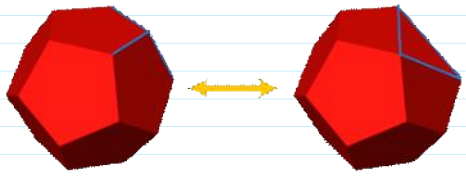


2. ∄ bdry triangle [Steinitz 1916, Grünbaum 1967, Gitler-Lichtenland 1986, Hass-Scott '94, Colin de Verdière '94, Nowik '09]

3. clear the caterpillar



Cor. Any 3-connected plane graph is the skeleton of a convex polyhedron.



Next time: Schnyder wood. straight-line embeddings