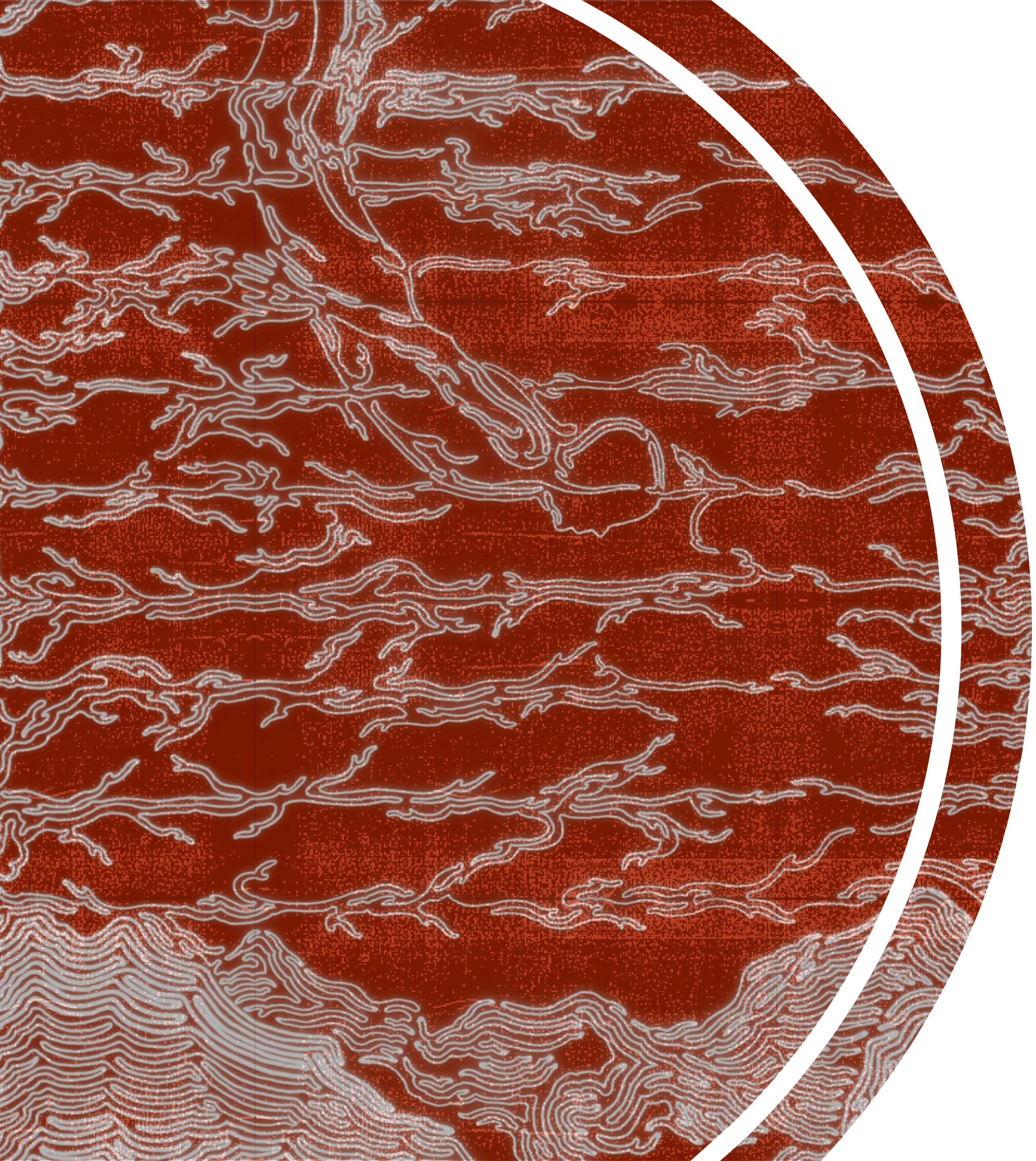


Introduction to Computational Topology

Hsien-Chih Chang

April 6, 2026

Surfaces (2d Manifolds)

What is a Surface?

- Formally, a surface (without boundary) is

A Haussdorff 2nd-countable topological space,
that is locally homeomorphic to the plane.

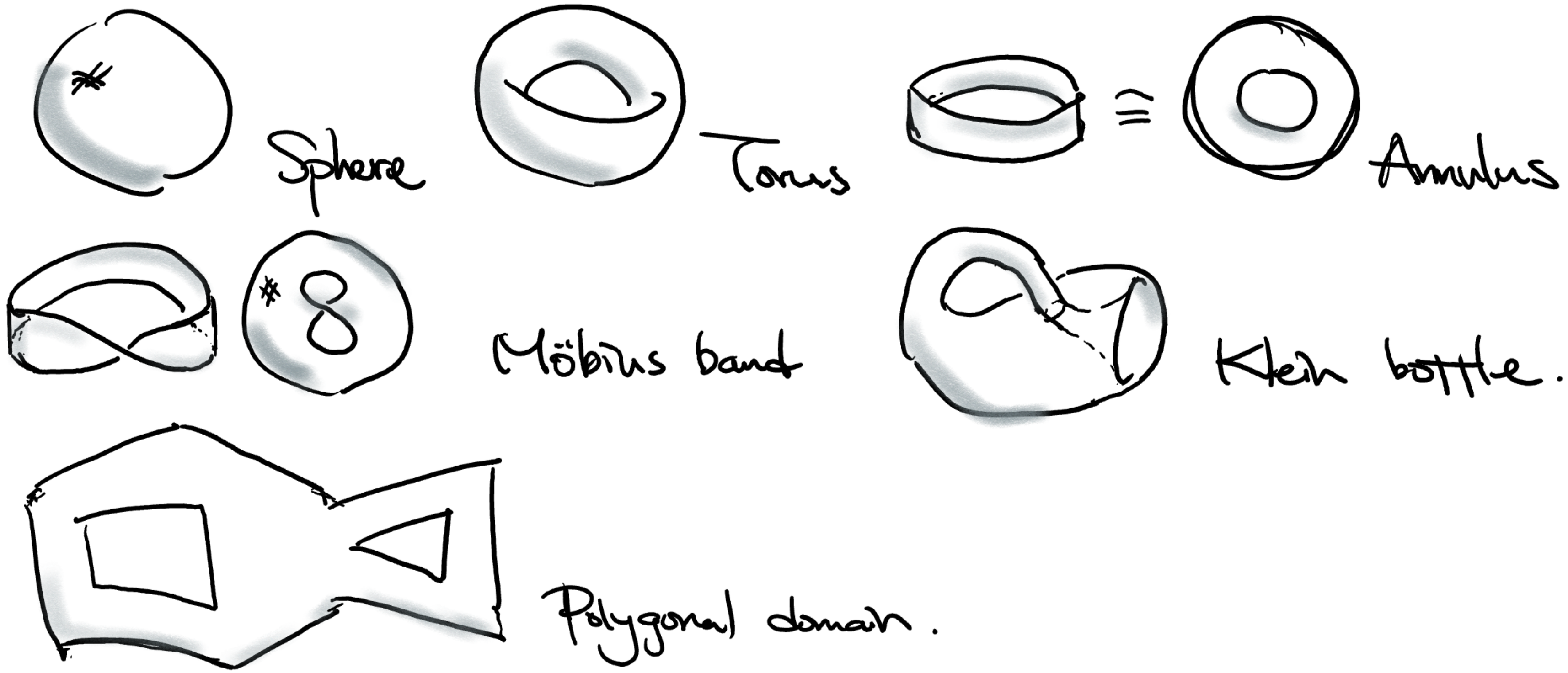
What is a Surface?

- Formally, a surface (with boundary) is

A Haussdorff 2nd-countable topological space,
that is locally homeomorphic to the plane or the half-plane.

Mythic Creature Exhibition

Mythic Creature Exhibition

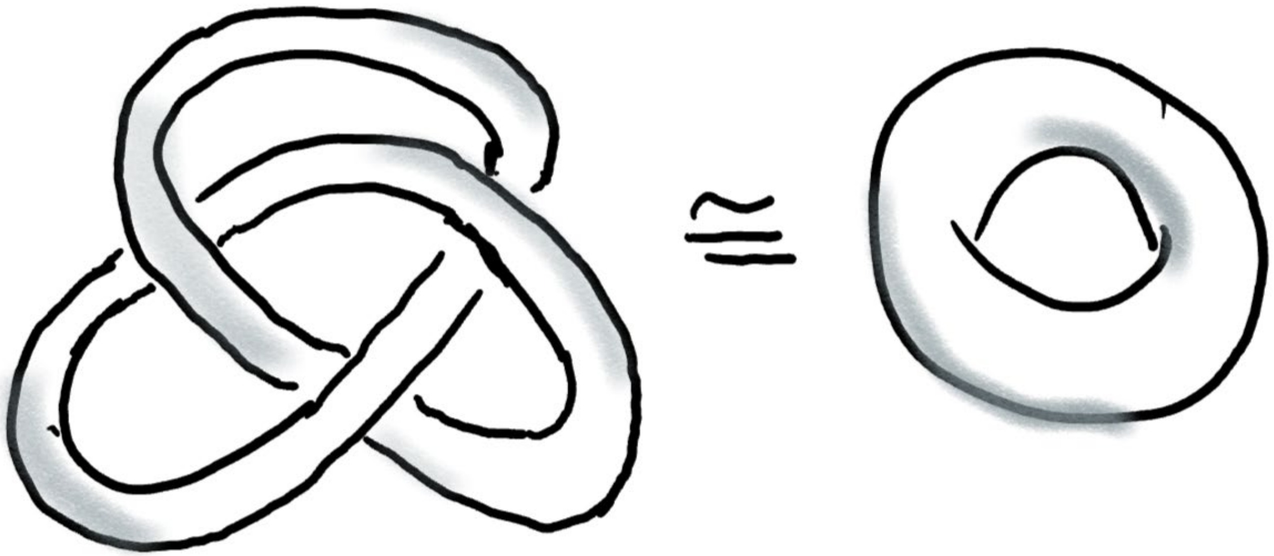


Tech-spec of the Fabric

- Bendable and stretchable

Tech-spec of the Fabric

- Bendable and stretchable
- Can phase-through itself



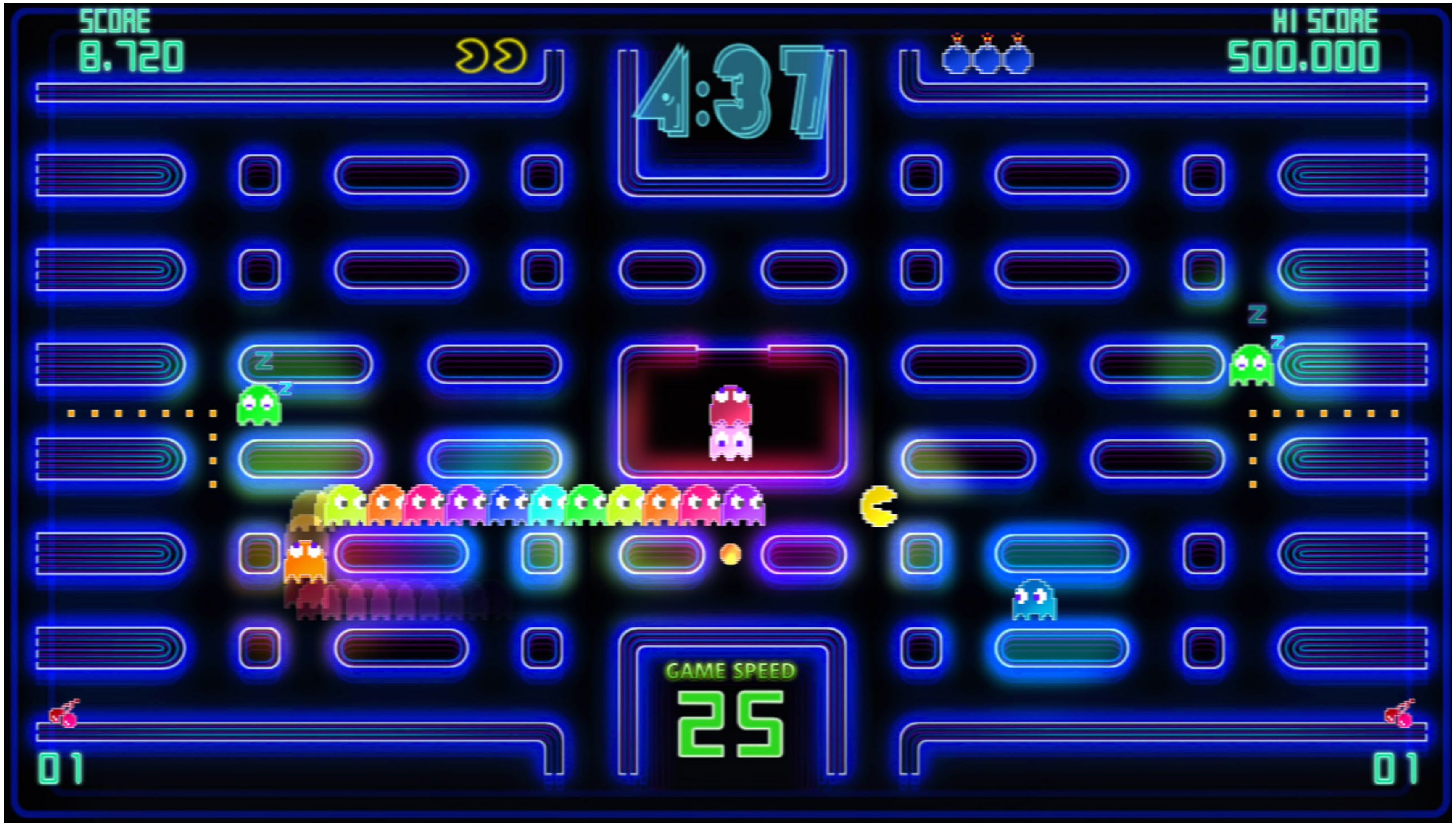
Tech-spec of the Fabric

- Bendable and stretchable
- Can phase-through itself
- NOT cuttable…

Tech-spec of the Fabric

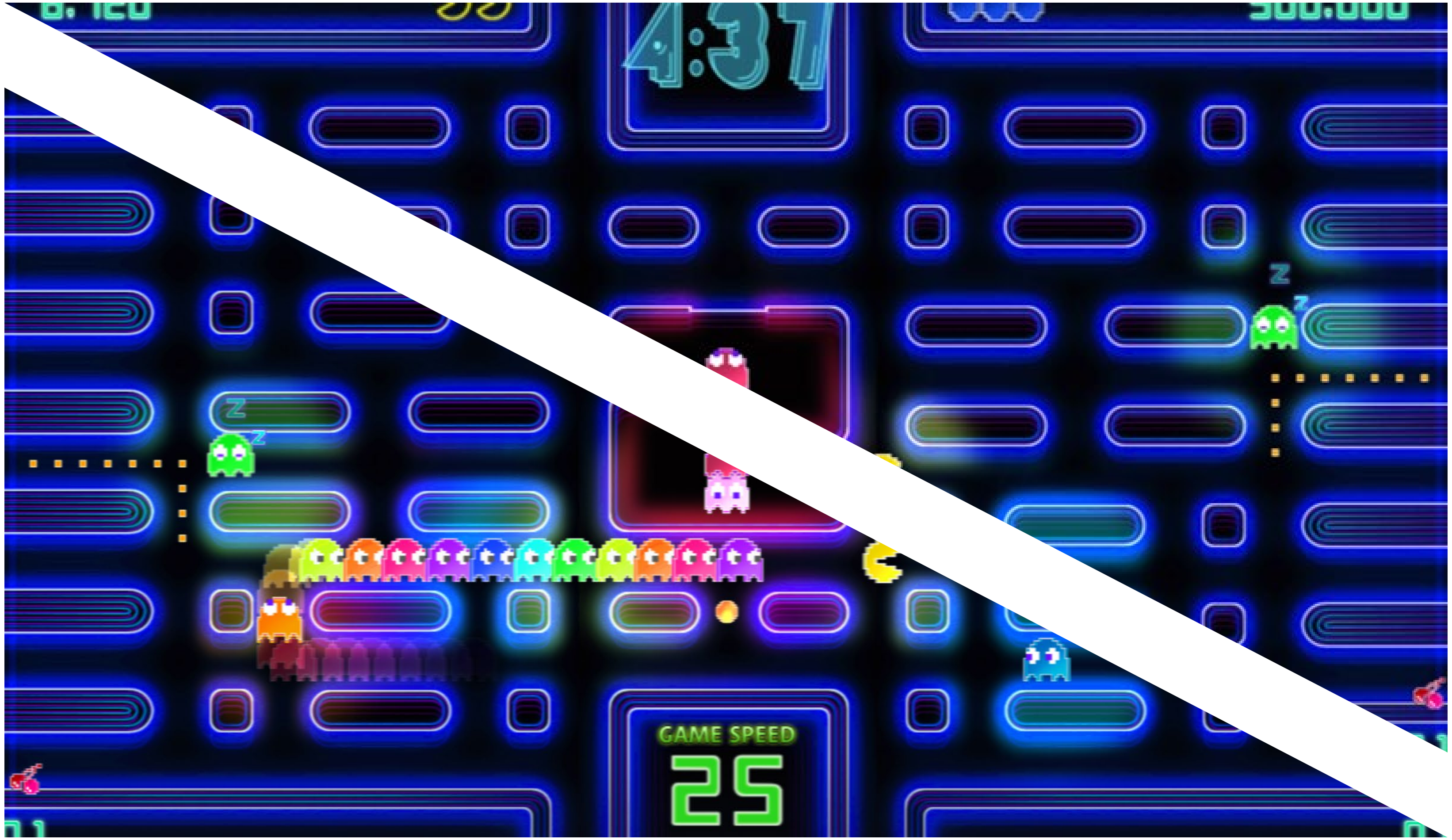
- Bendable and stretchable
- Can phase-through itself
- NOT cuttable…
  …unless you glue it back

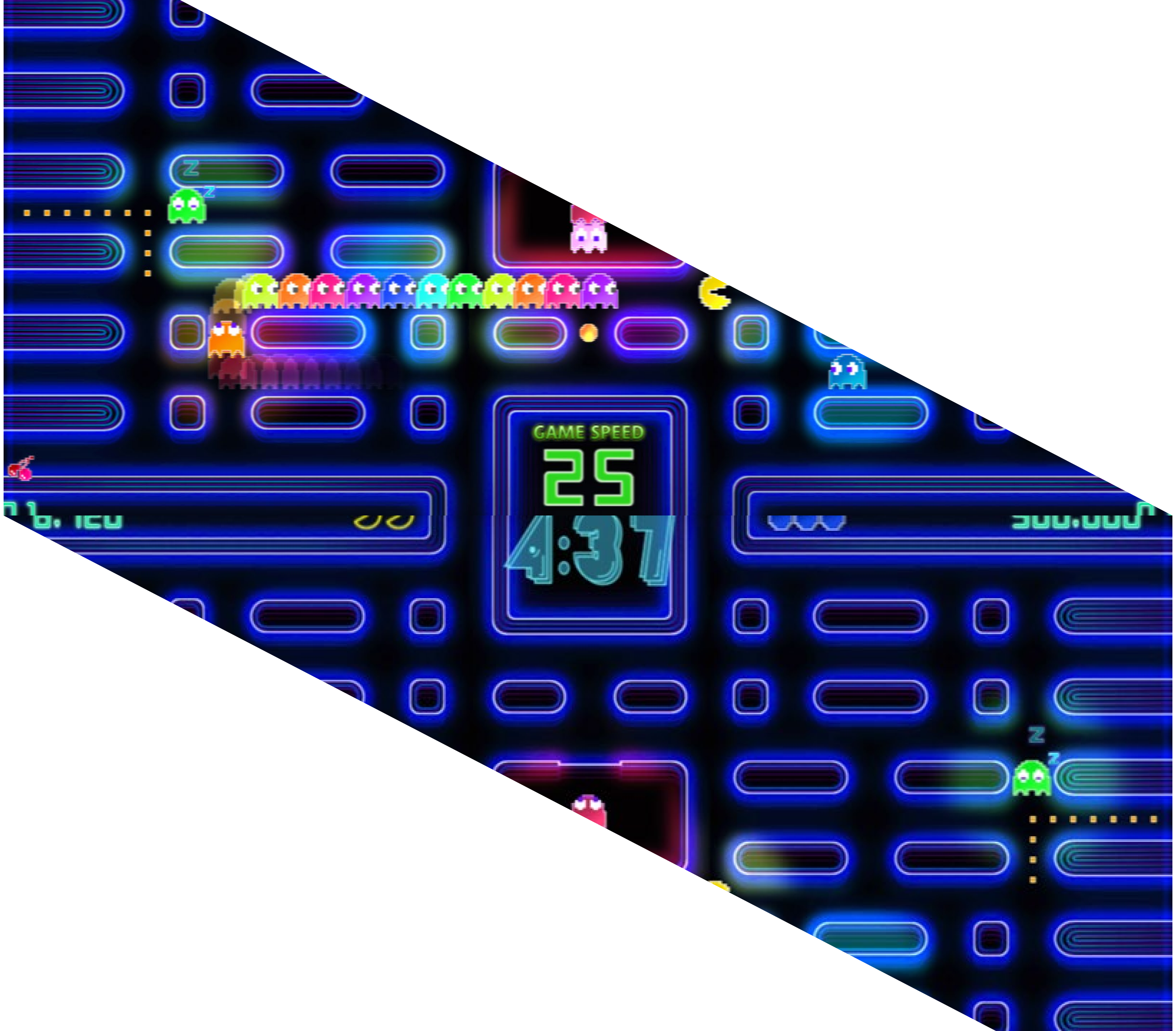
Example: Pacman Space



Example: Pacman Space

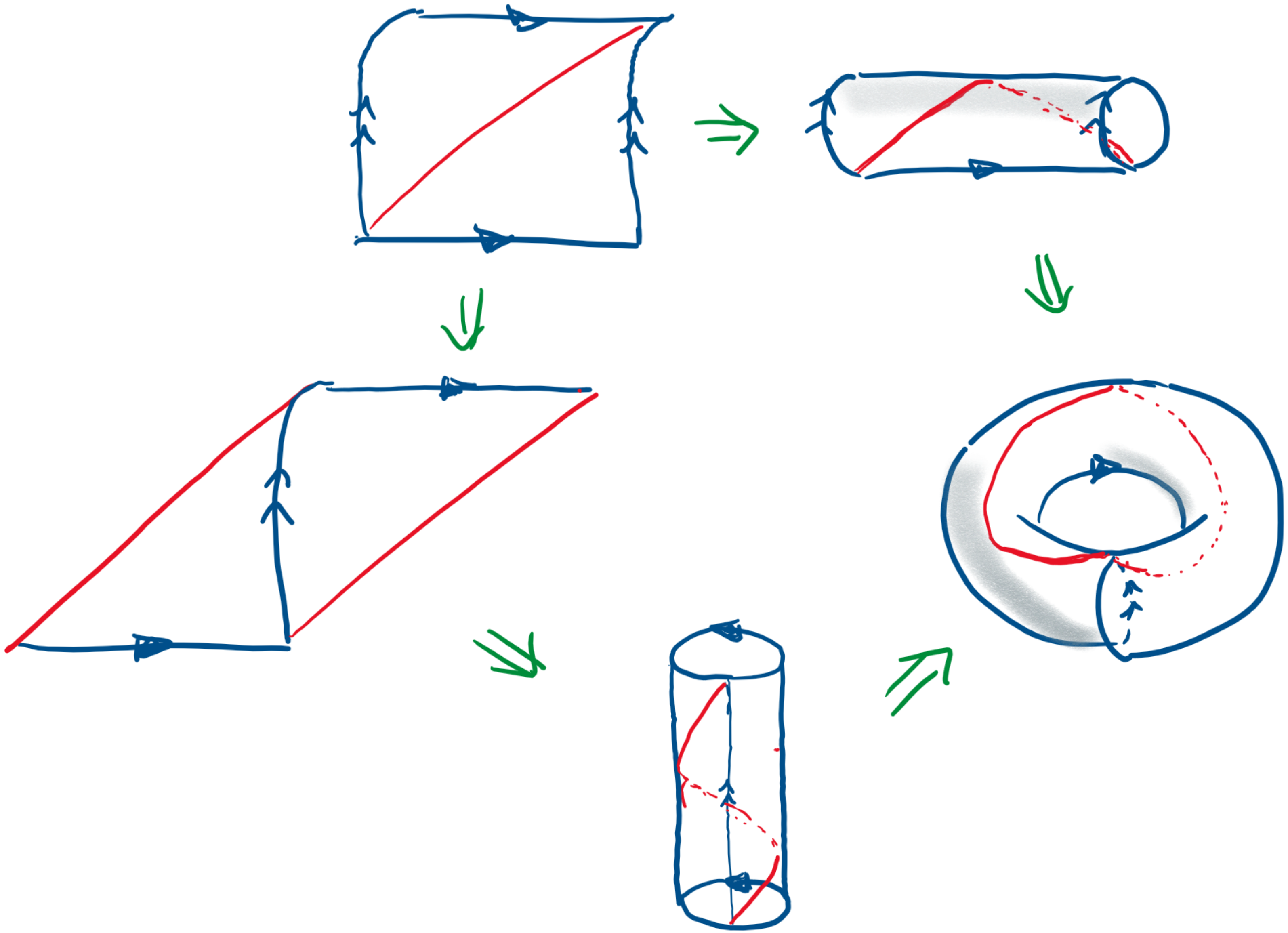
Example: Pacman Space





Example: Pacman Space

Example: Pacman Space



Exercise: What is this Surface?

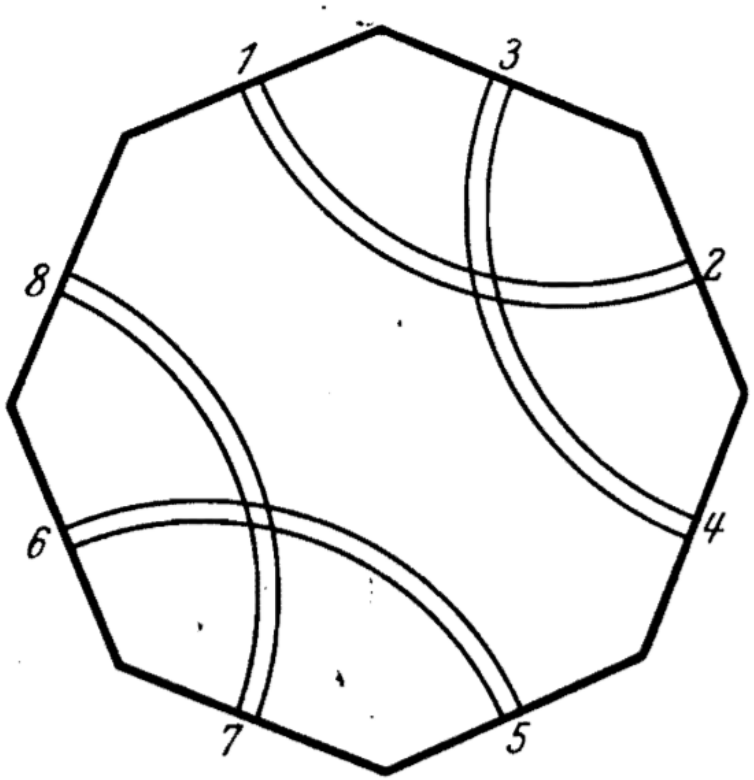


Fig. 286a

Exercise: What is this Surface?

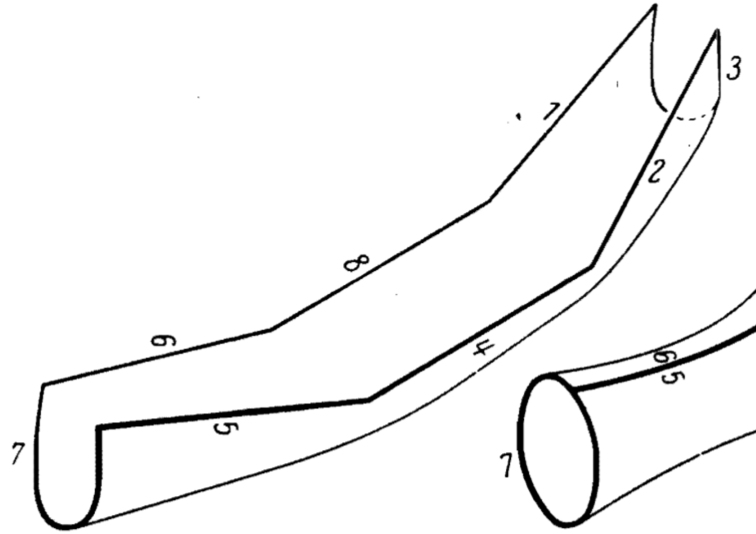


Fig. 286b

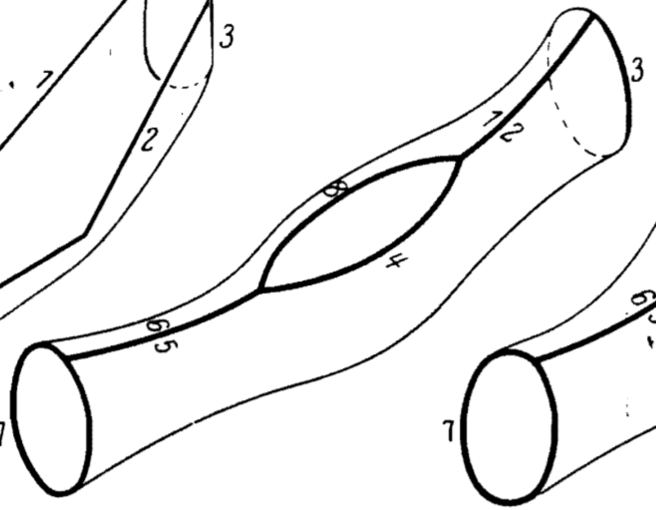


Fig. 286c

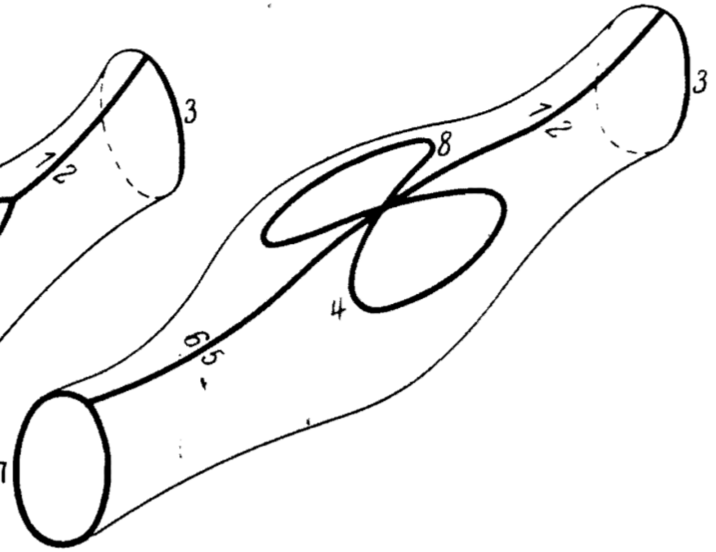


Fig. 286d

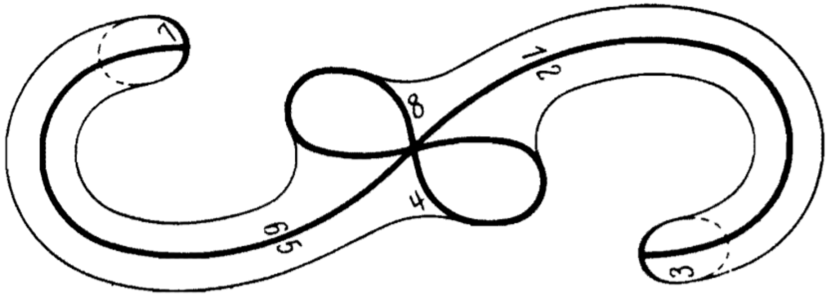


Fig. 286e

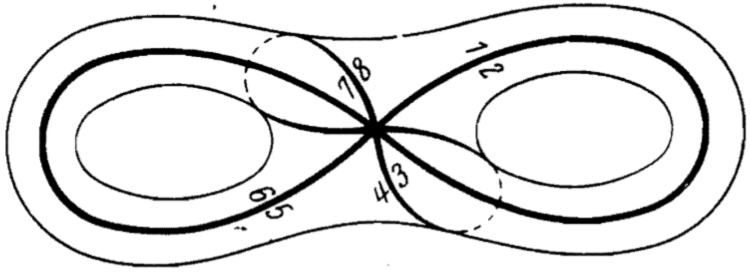
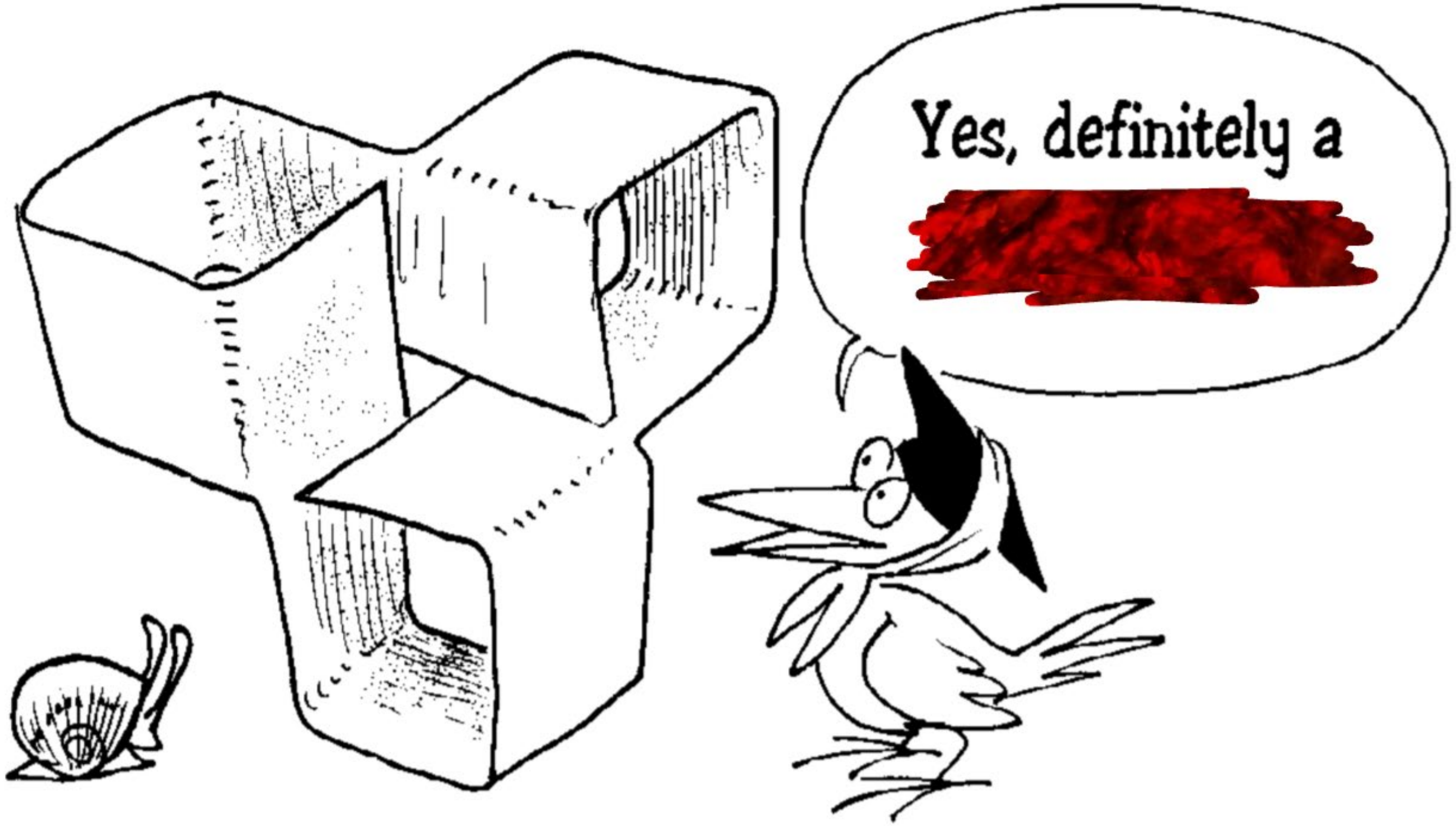
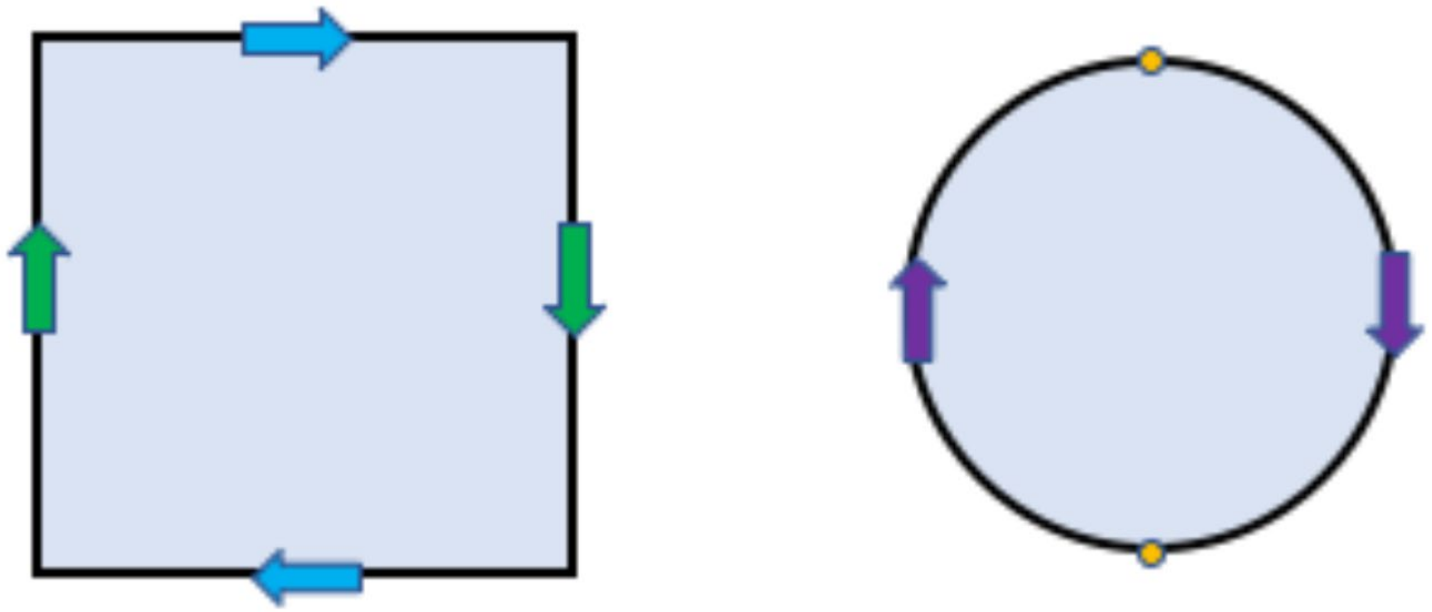


Fig. 286f

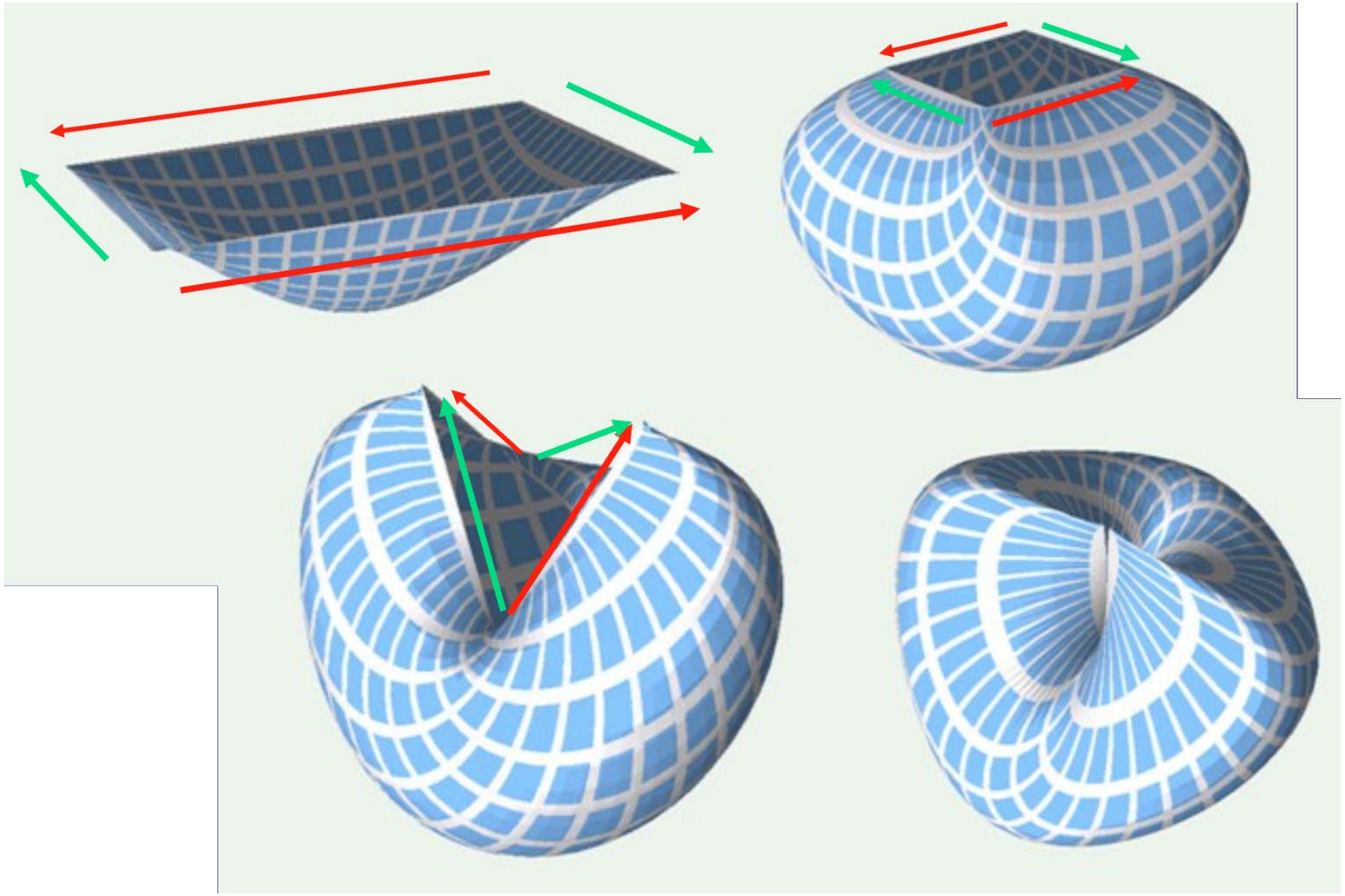
Exercise: What is this Surface?



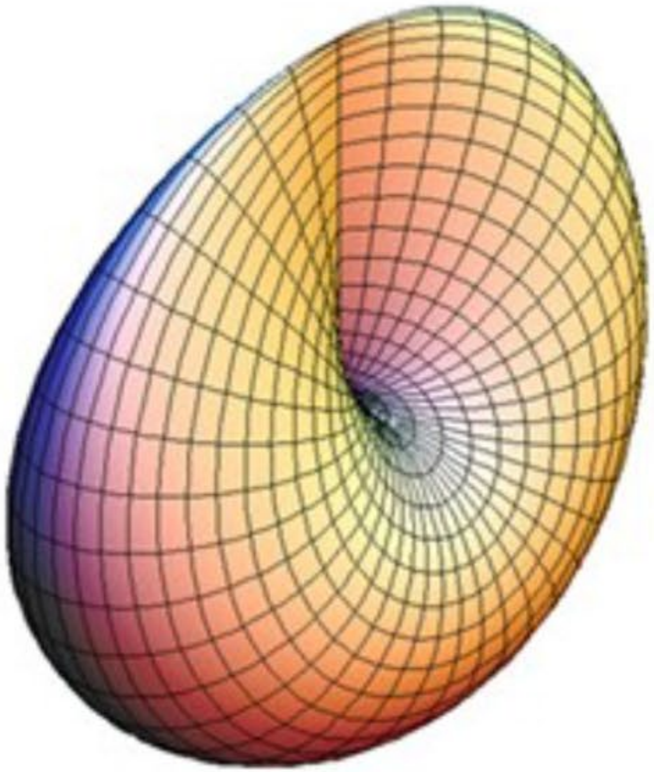
Exercise: What is this Surface?

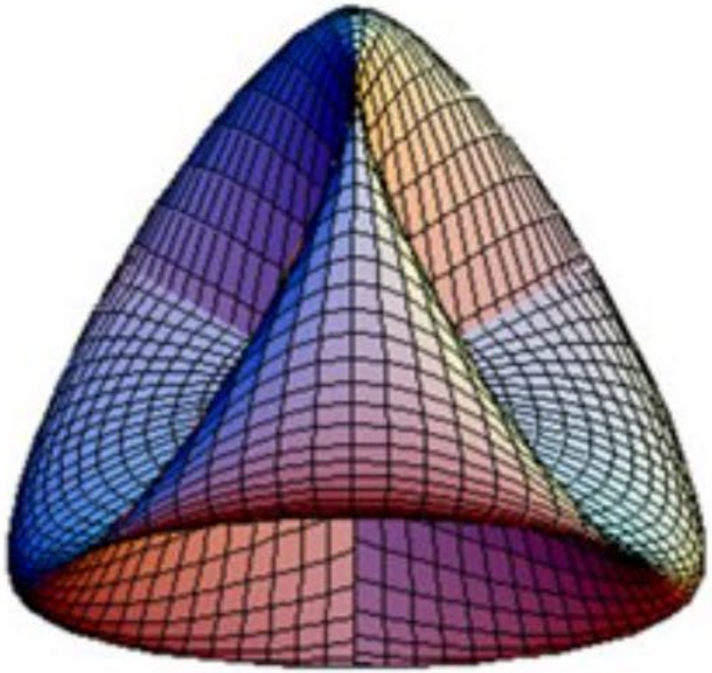


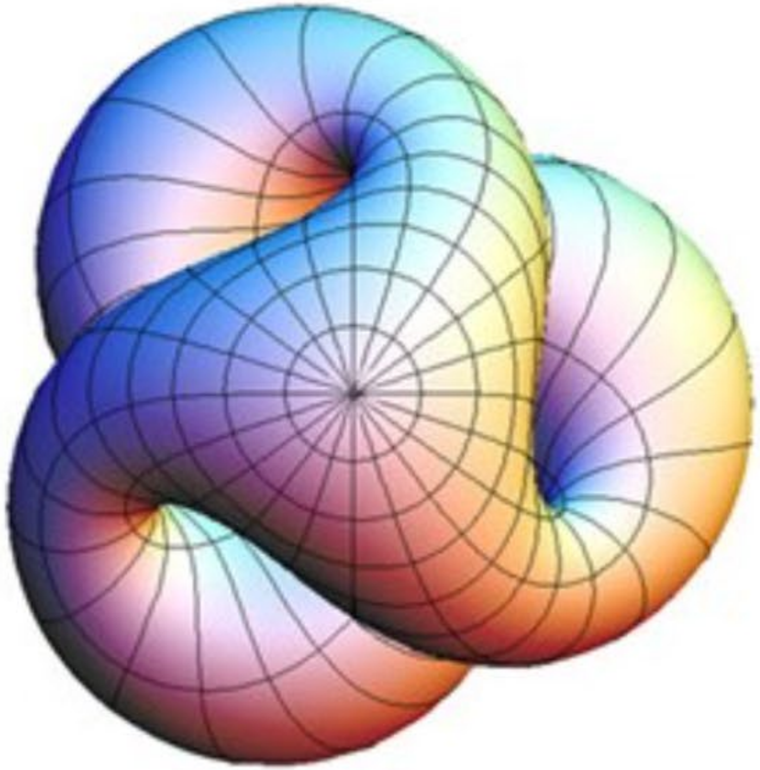
Exercise: What is this Surface?



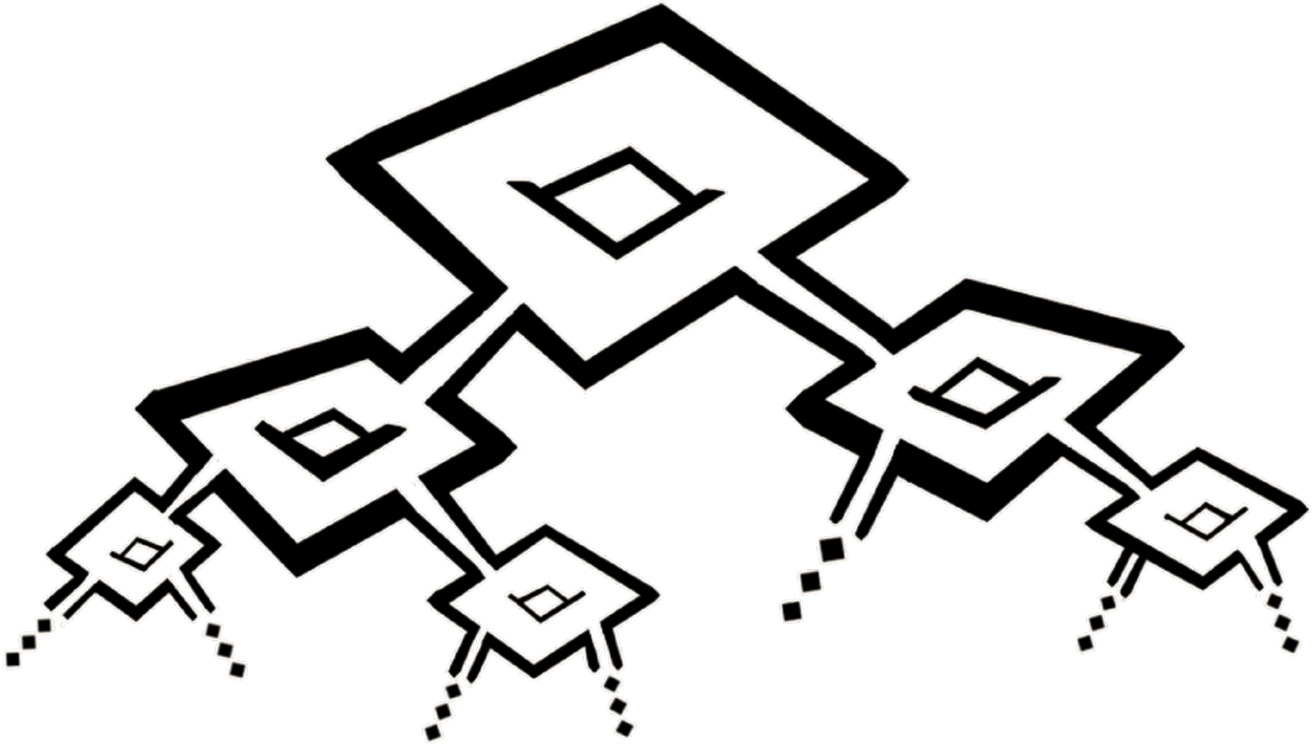
Exercise: What is this Surface?







Can we get all surfaces through cut-and-paste?



Surface Classification

[Möbius 1861] [Dehn-Heegaard 1907] [Radó 1925]

Every connected surface is homeomorphic to the following:

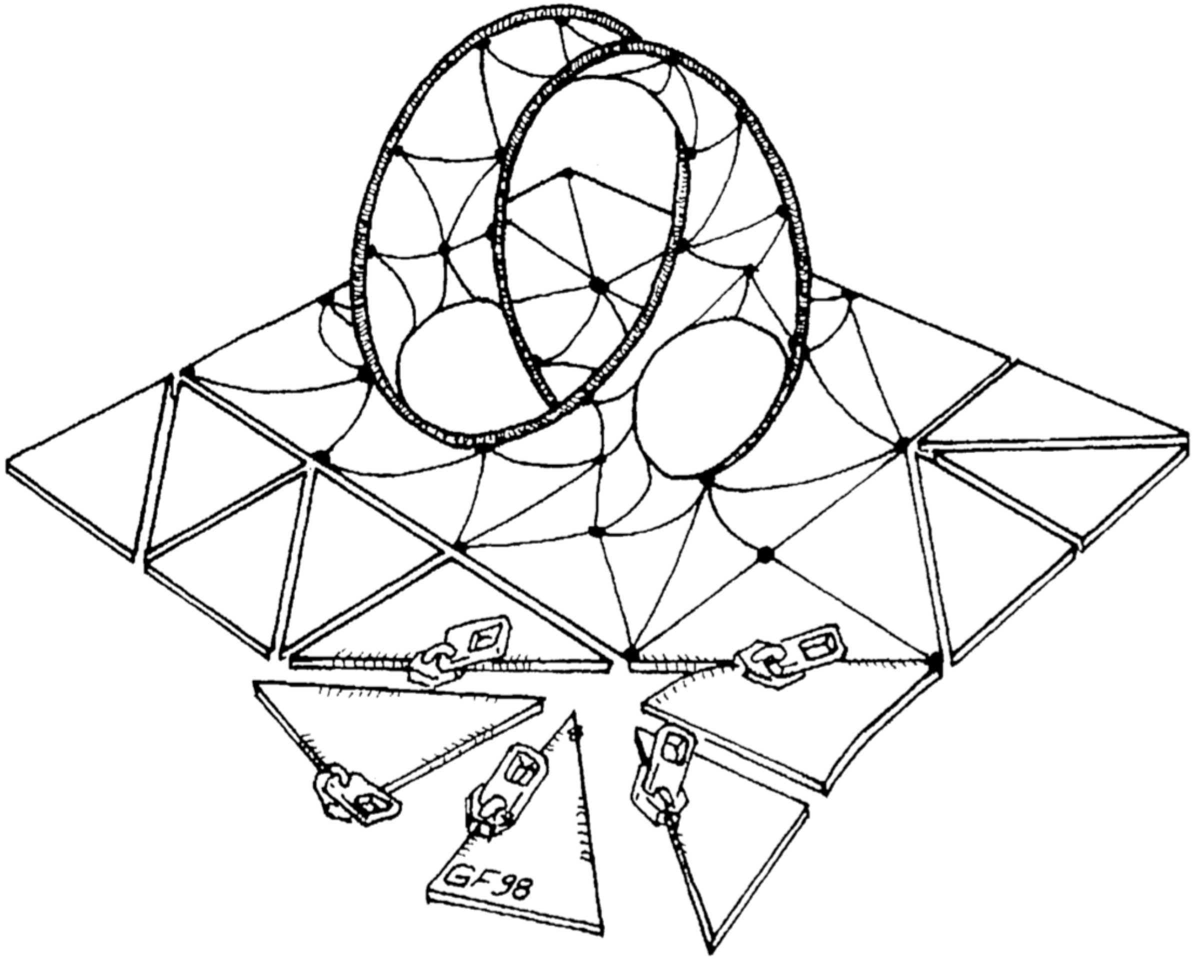
(plus boundaries)

- Sphere with g handles $\Sigma(g, 0)$
- Sphere with r cross-caps $\Sigma(0, r)$

Theorems we secretly assumed

- Triangulation Theorem [Kerékjártó-Radó 1925]
  - Any surface can be cut into triangles
- Refinement Theorem [Moise 1977]
  - Any two triangulations have a common refinement

Conway's ZIP Proof



Conway's ZIP Proof

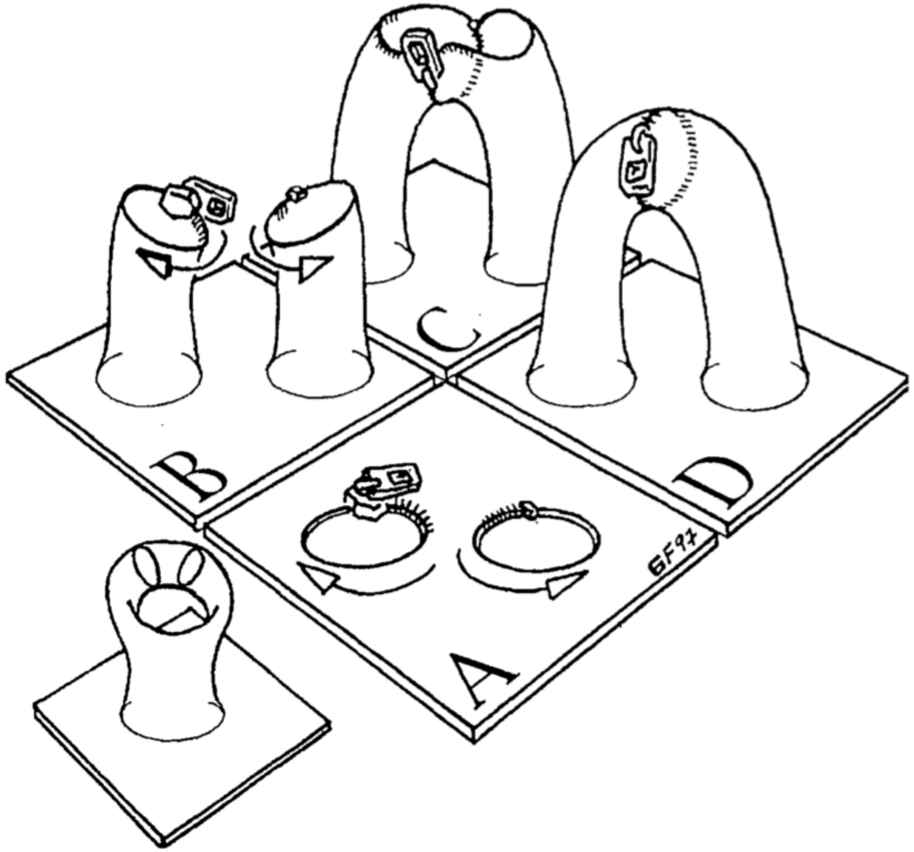


Figure 1. Handle

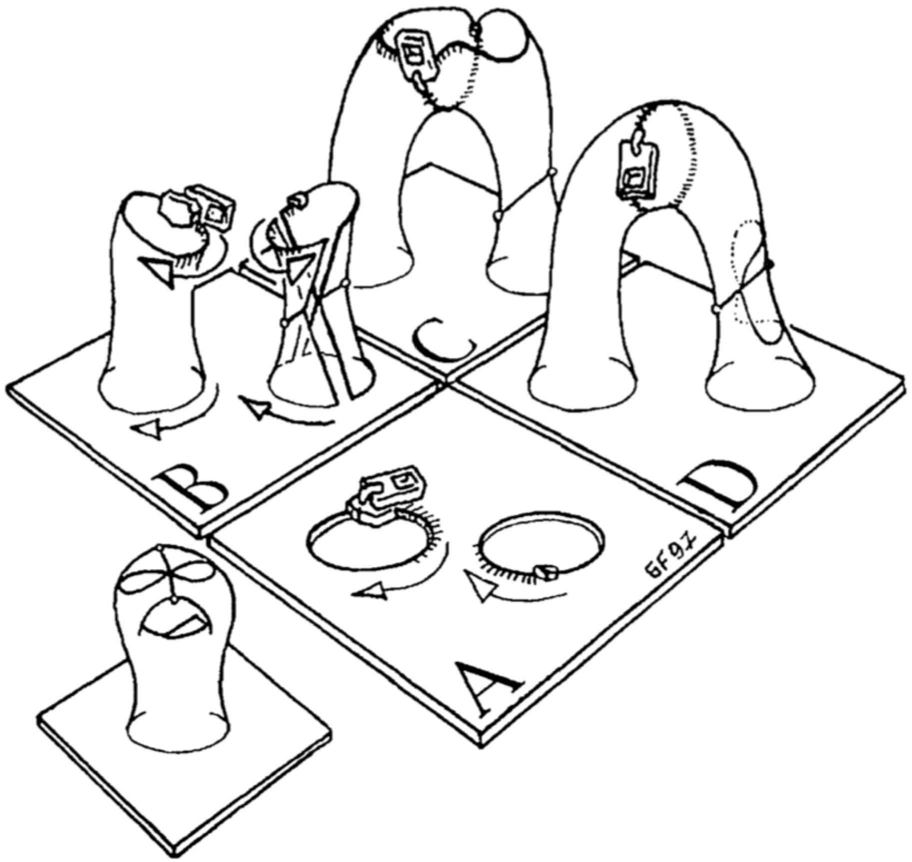


Figure 2. Crosshandle

Conway's ZIP Proof

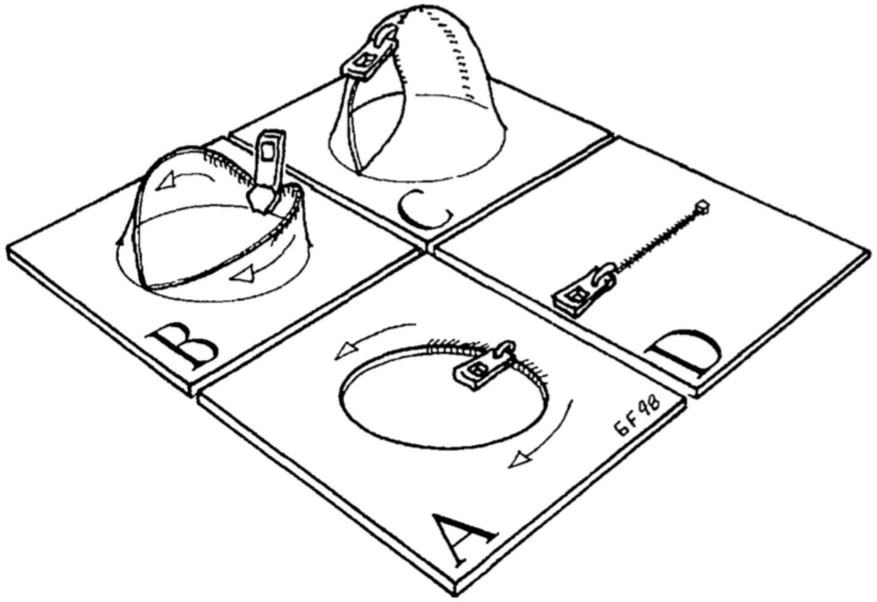


Figure 3. Cap

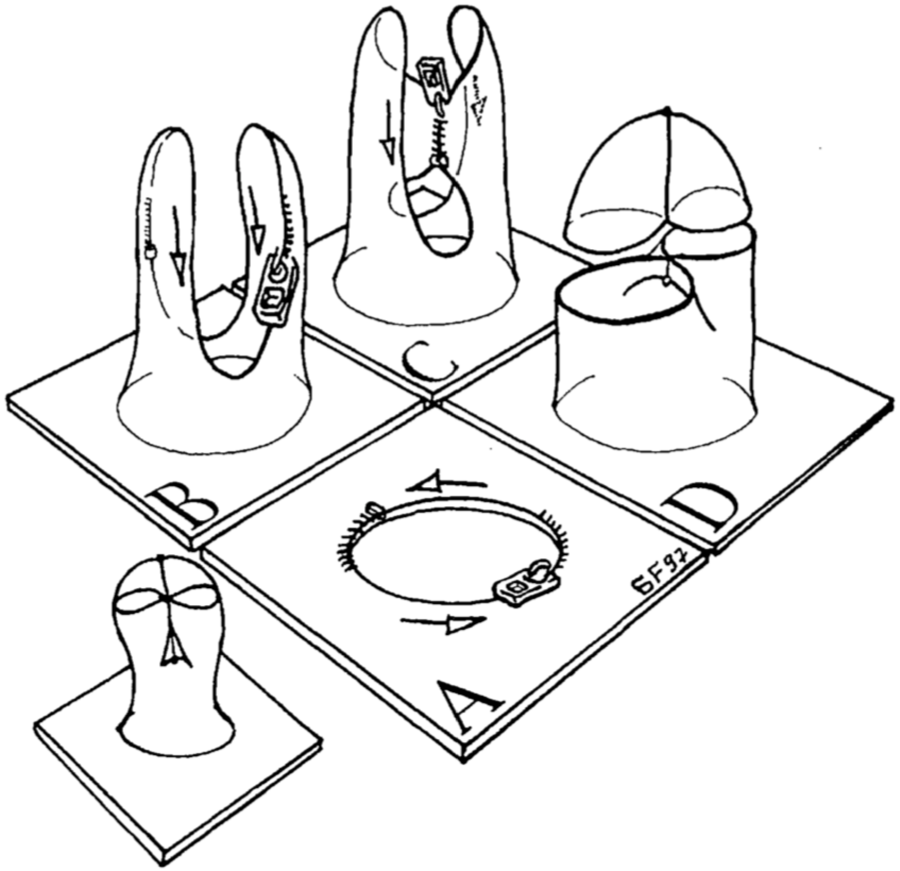
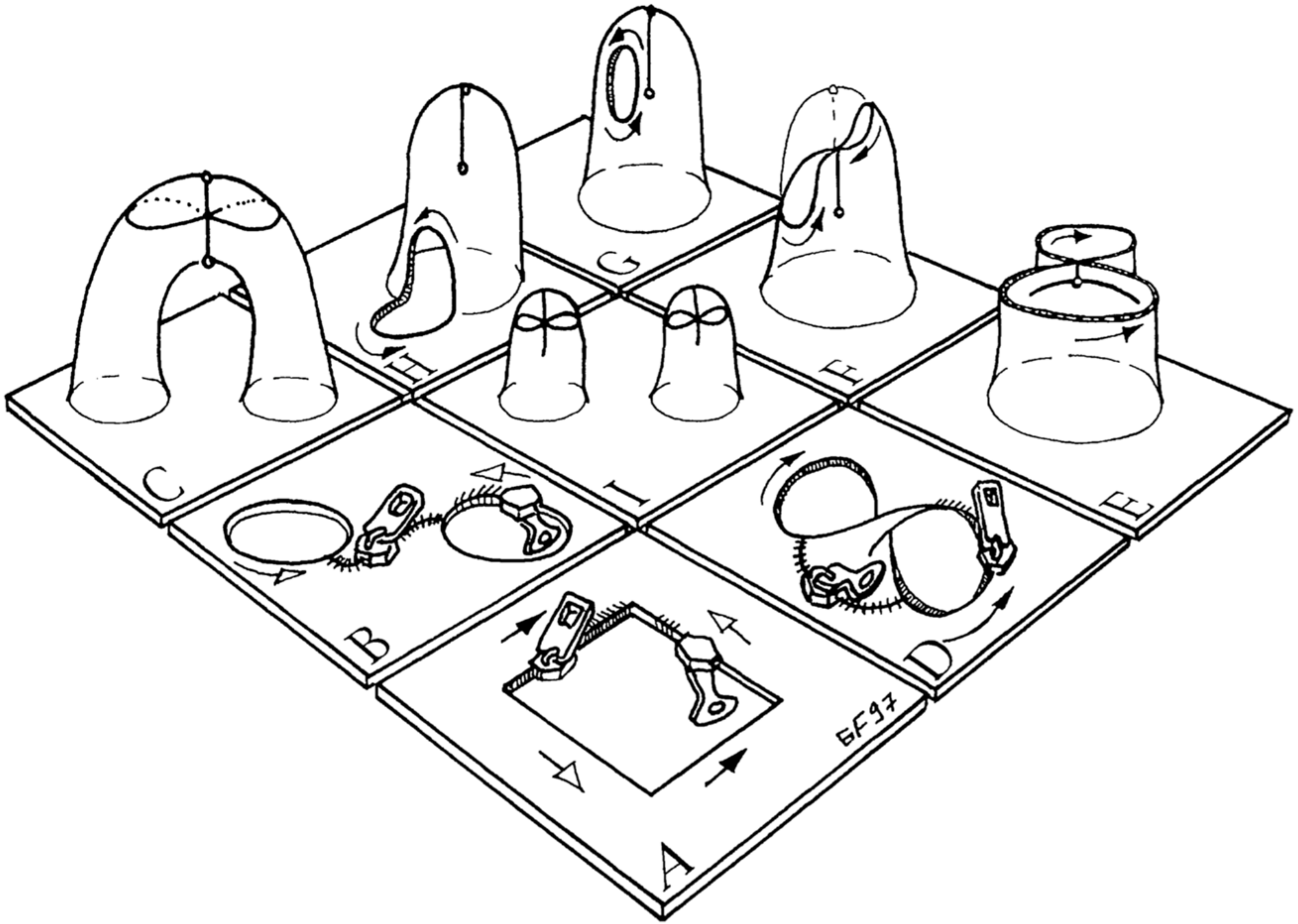
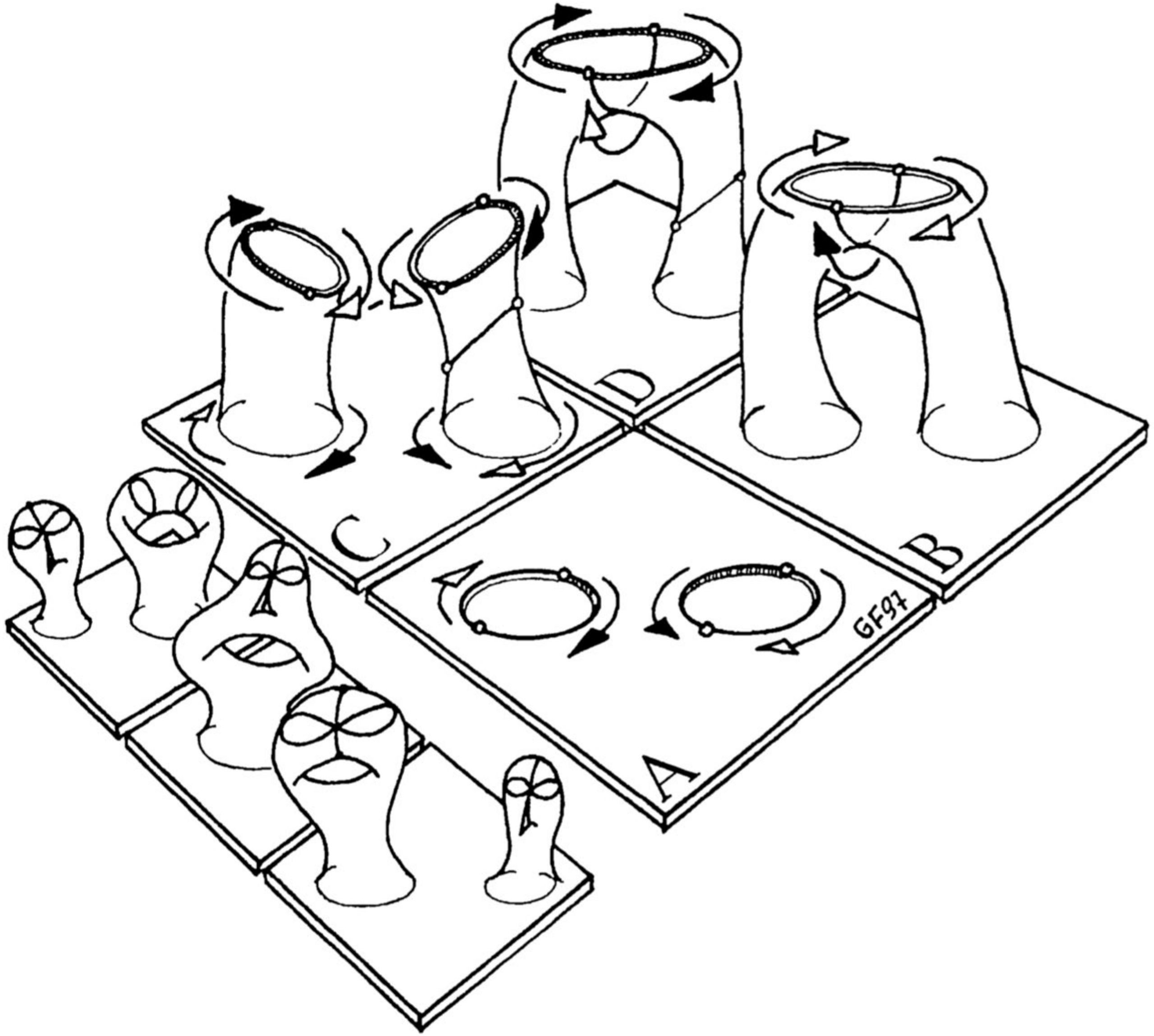


Figure 4. Crosscap



K=P#P

Exchanging two cross-caps for a cross-handle



T#P=K#P

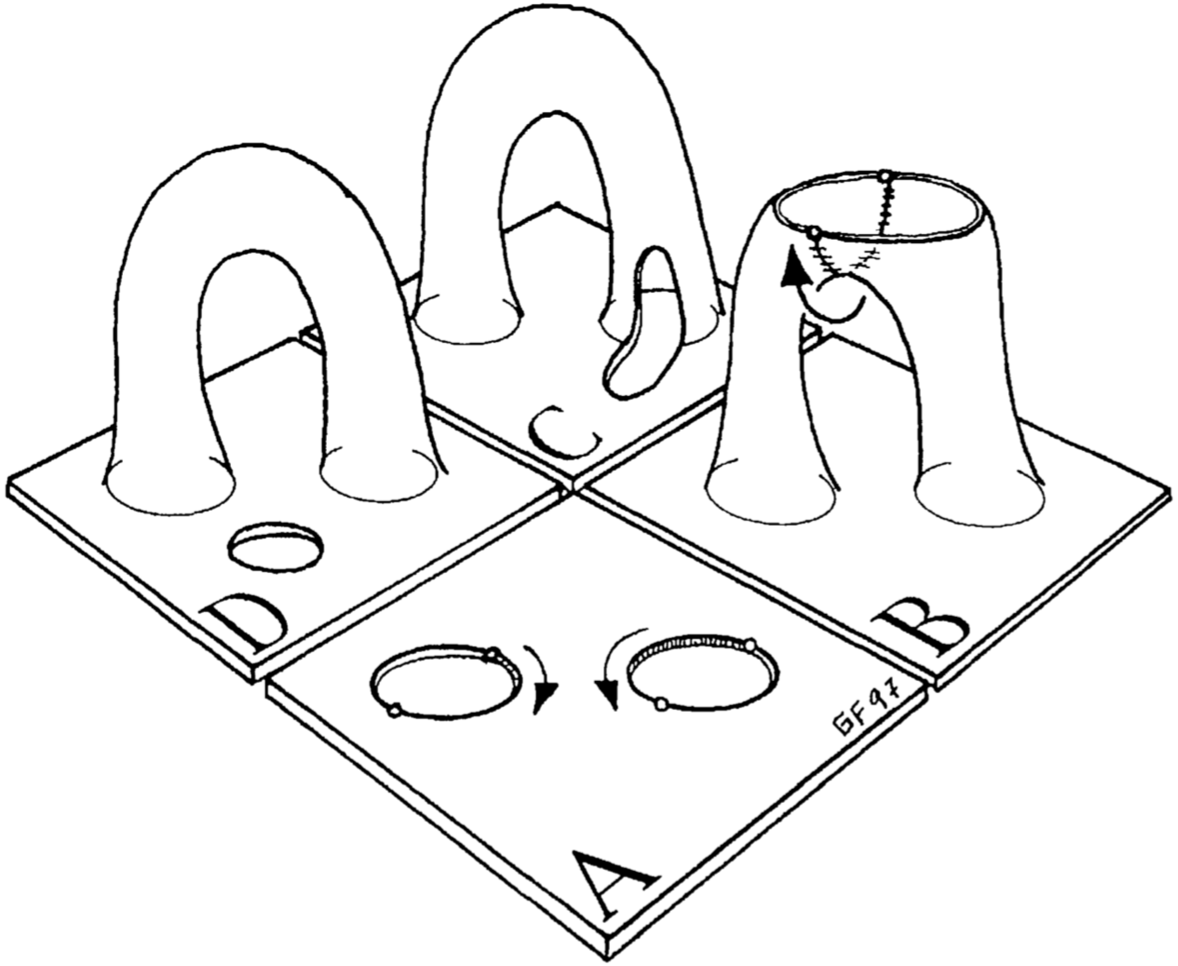
Handles and cross-handles are the same when cross-caps are presented

Trading

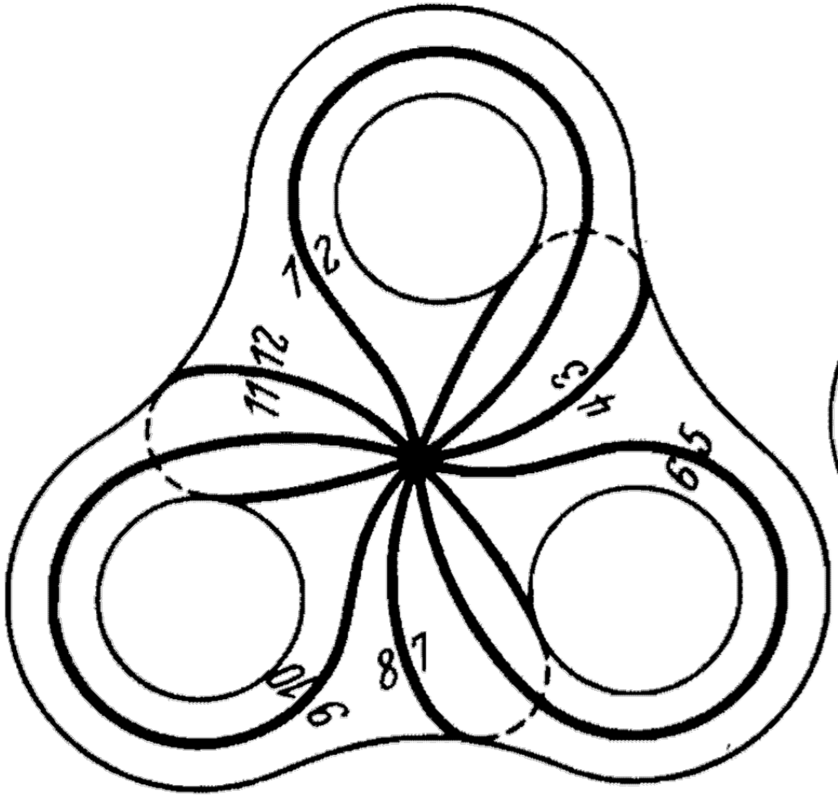
- When cross-handles or cross-caps are presented
  - Turn all handles and cross-handles into cross-caps

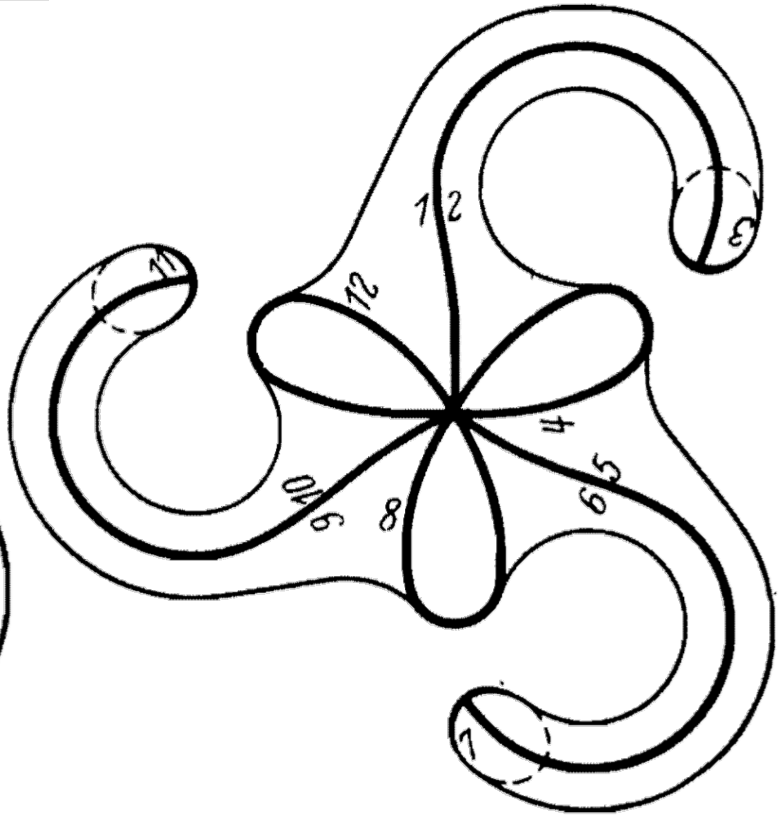
- Otherwise, only handles exist

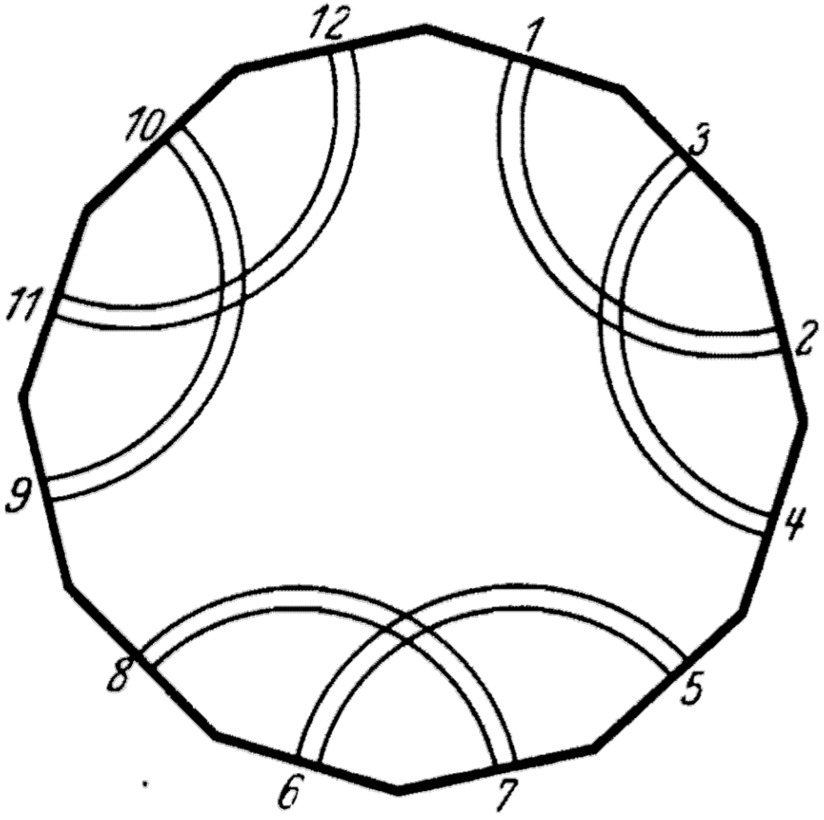
Dealing with Boundaries



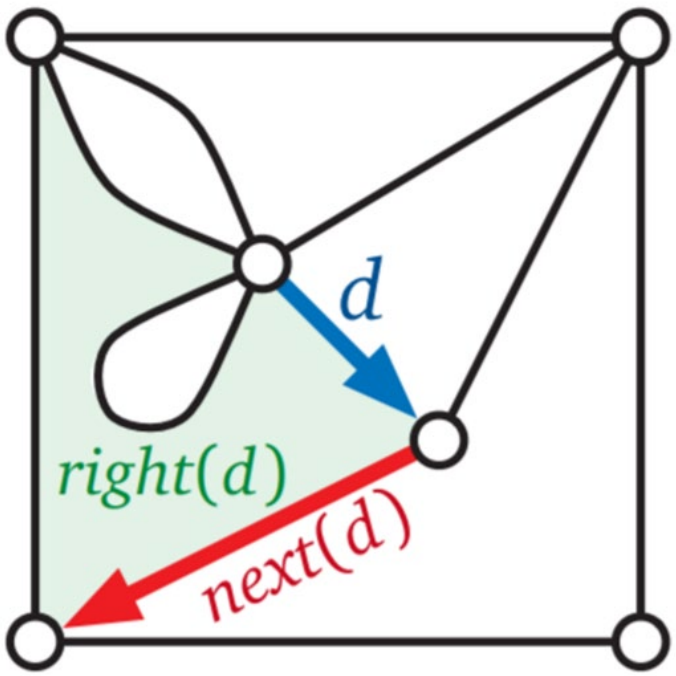
Treat Cutting-lines as Graphs

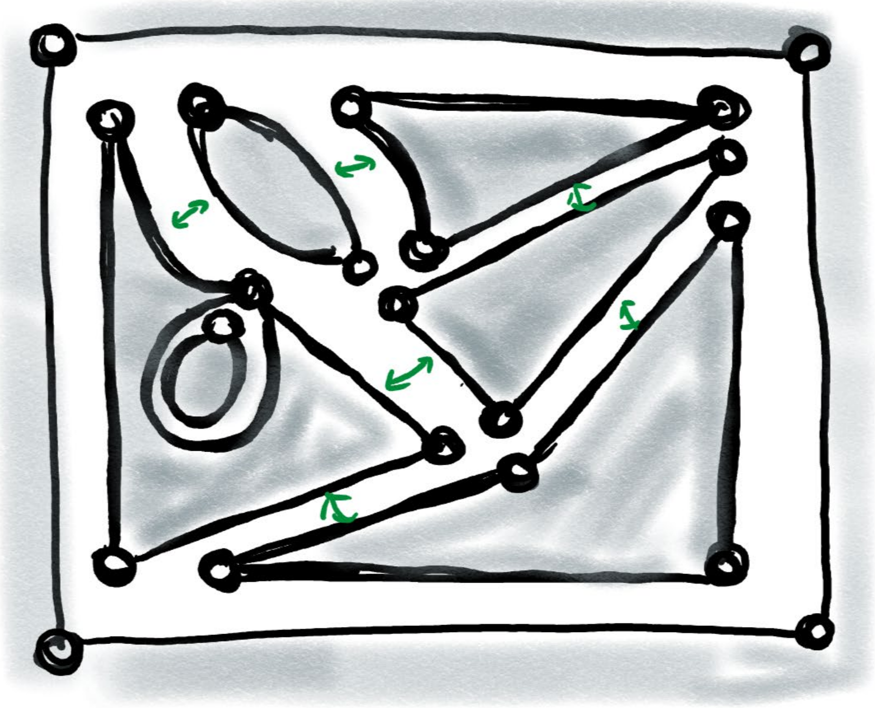


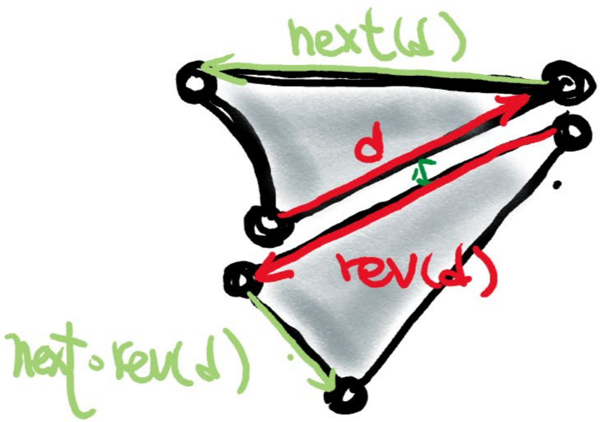




Polygonal Schema IS Rotation System







Q. Does Euler's Formula Hold for Surface Graphs?

Next time.

Surface hard to visualize?

The space is weird?