0401

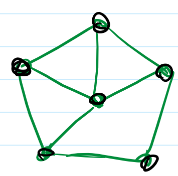
Thursday, September 19, 2024 10:30 AM

Administrivia.

- Find course webpage, Join Gradescope/Slack!
- HW 0. due next Monday
- Put more structure into the course
  - presentation
  - scribe / participation
  - homework sets.

$\boxed{\text{Planar graphs have ... Duality \& Euler Formula}}$

- Planar graphs as polygonal schema.



Euler characteristic $\chi := V - E + F$

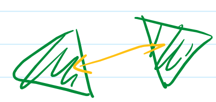
$6 - 9 + 5 = 2$

Inclusion-Exclusion: $\chi(A \cup B) = \chi(A) + \chi(B) - \chi(A \cap B)$.

$1 + 1 - 1 = 1$

$\chi(\triangle) = 3 - 3 + 1 = 1$

$\chi(|) = 2 - 1 + 0 = 1$



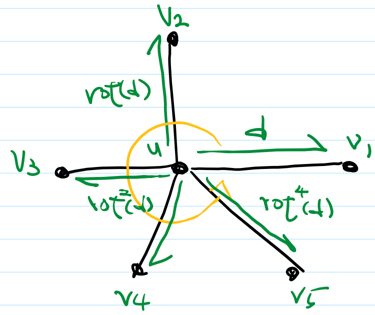
Implicit Assumption:

1. Forms a topological sphere
2. Every face is cellular
3. Has a shelling order

- Planar graphs as rotation system.

v_2

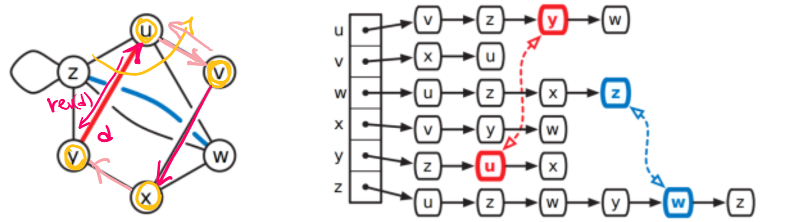
• Planar graphs as rotation system.



~~G = (V, E)~~ ...

- ~~vertices V~~, darts D
- rev() $rev : D \to D$
- rot() / succ() $rot : D \to D$
- $head : D \to V$

$tail(d) = head \circ rev(d)$

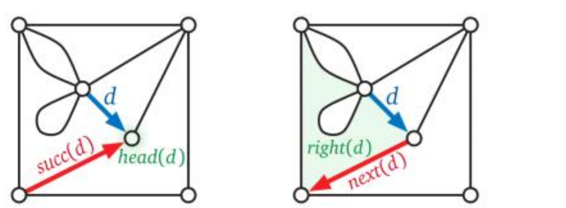


An incidence list representation of a graph, with the dart records for two edges emphasized. For clarity, most reversal pointers are omitted.

- next() $next : D \to D$ $next(d) := rot \circ rev(d)$

orbit of next() are faces!

-



The successor and dual successor of a dart in an embedded planar graph.

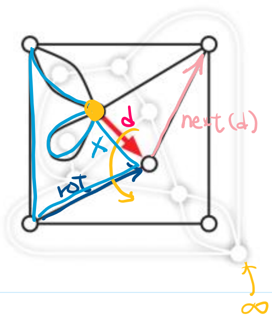
Polygonal schema = Rotation system.

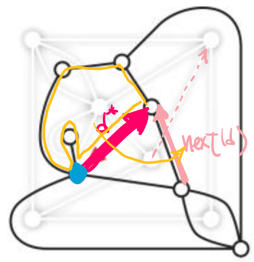
• Dual rotation system.

$(rev, rot, D) \longleftrightarrow (rev, next, D)$

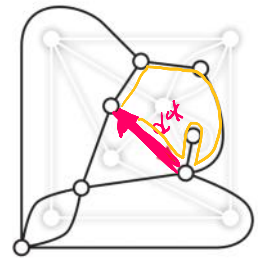
◦ clockwise ⟷ ccw

(rev, rot, D) ⟵ (rev, next, D)





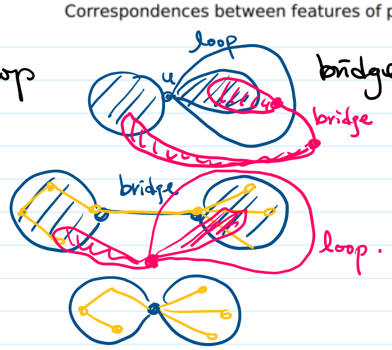
- clockwise ⟺ ccw
- right(d) ⟺ tail(d*)
- tail(d) ⟺ right(d*)



primal G	dual G^*	primal G	dual G^*
vertex v	face v^*	empty loop	spur
dart d	dart d^*	loop	bridge
edge e	edge e^*	cycle	bond
face f	vertex f^*	even subgraph	edge cut
$tail(d)$	$left(d^*)$	spanning tree	complement of spanning tree
$head(d)$	$right(d^*)$	$G \setminus e$	G^* / e^*
$succ$	$rev \circ succ$	G / e	$G^* \setminus e^*$
clockwise	counterclockwise	minor $G \setminus X / Y$	minor $G^* \setminus Y^* / X^*$

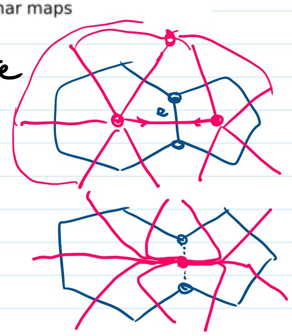
Correspondences between features of primal and dual planar maps

loop



bridge

delete



contract

- Spanning tree algorithm

SpanningTree(G^*):

for any edge e^* in G:

 if e^* is a ~~loop~~ bridge:

  ~~delete~~ contract e^*

 if e^* is a ~~bridge~~ loop:

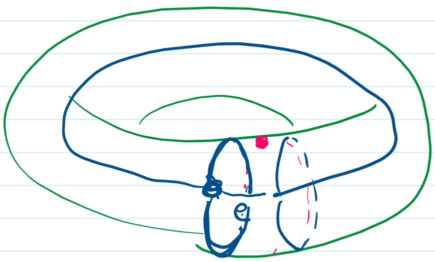
  ~~contract~~ delete e^*

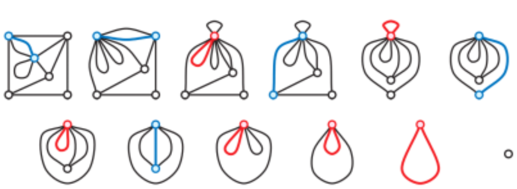
 o.w. ~~delete~~ contract or ~~contract~~ delete e^*

return all contracted edges

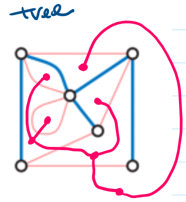
$$E = \underbrace{T}_{\text{tree}} \sqcup \underbrace{C}_{\text{cotree}} \sqcup L^2$$

"Tree-cotree decomposition"





Computing a spanning tree of a graph.



Proof of Euler Formula.

$$E = (V-1) + (F-1) = V + F - 2$$

$$V - E + F = 2 - \ell \quad \blacksquare.$$

Planar graphs have ... Separators

Separator

Cycle separator

Thm. [Miller'86]. [Lipton–Tarjan'79] Planar graphs have (cycle) separator.

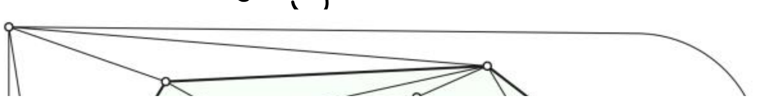
Start w/ a triangulation

- level separator. L_ℓ.
- coedge separator cycle(T, xy)

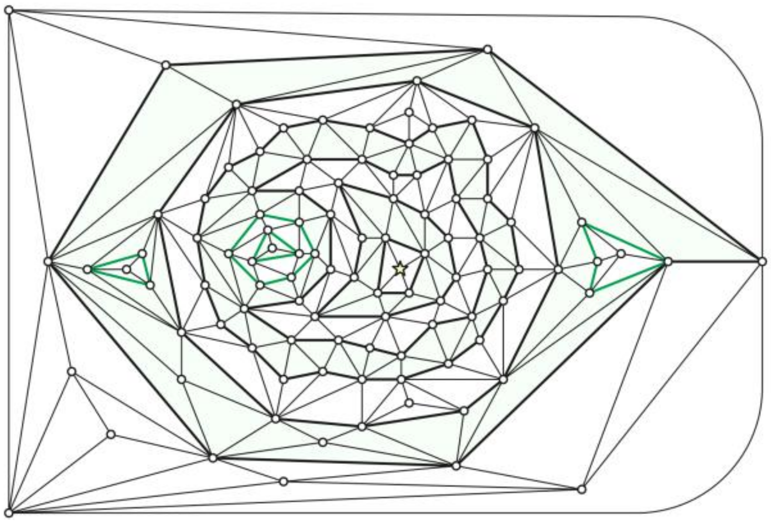
Prop. level separator containing the "median vertex" is balanced.

Prop. coedge separator using the center coedge in T^* is balanced.

Dual BFS (layering).



$lv(f)$



$\ell_v(f)$

$R_{\le i}$

C_i

Lemma.

- the "boundary" of $R_{\le i}$ is a simple cycle, C_i
- all C_i's are pairwise vertex-disjoint.
- cycle(T, xy) intersects each C_i at most twice.

Claim.

Claim.

[Frederickson '87]

Good r-division: n-vertex plane graph G.

- Chop G into $O(n/r)$ pieces
- each piece has size $\le r$.
- # bdry vertices per piece is $O(\sqrt{r})$
- # holes per piece is $O(1)$

Idea: Iteratively cycle through 3 kinds of weights.

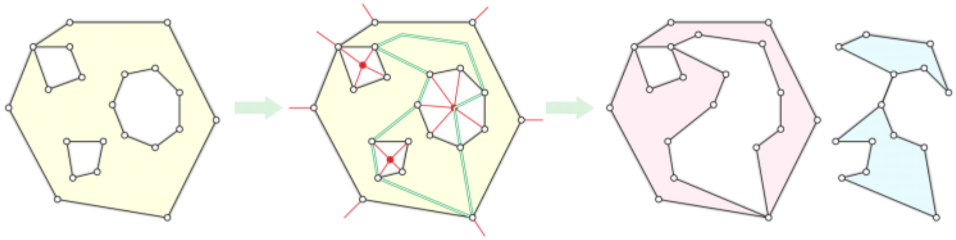


Figure 3: A region with three holes, a cycle separator for the triangulated region, and the resulting smaller regions.

Next time: curvature, Gauss-Bonnet, Cauchy Rigidity Thm.