All About Planar Graphs

(Topics in Computational Topology)
COSC 249 (Fall 2024)

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Course Description

Planar graphs are among the most well-studied graph classes since the beginning of graph theory and algorithm design. Planar graphs can manifest in computational problems as networks, meshes, or terrains that are geometric and low-dimension in nature, which unsurprisingly find many applications in computer graphics, visualization, vehicle routing, and shape analysis.

This graduate-level seminar-style course is aimed to introduce a vast array of tools for planar graphs, ranging from combinatorial to geometric to spectral to algorithmic and beyond. Naturally, due to the sheer amount of work and literature in the area, the topics covered in this class will be biased towards the interest and expertise of the instructor.

Course Objectives

The course serves as an opportunity for people interested in planar graphs and their applications to learn about the various tools and techniques developped, both classical and recent.

Prerequisite

Students are assumed to have the math maturity to read and write proofs and establish basic properties of newly encountered mathematical structures. Knowledge in graph theory (*COSC 30 Discrete Math* or *Math 38 Graph Theory*) and linear algebra (*Math 22*, *Math 24* or *COSC 70* or Instructor Permission) are required as prerequisite. Knowledge in algorithms (*COSC 31 Algorithms*) are strongly recommended.

Textbook

There is no required textbooks. The class touches upon various topics in combinatorial topology, graph minor theory, topological graph theory, graph drawing, spectral graph theory, and graph algorithms; many topics are based on currect active research in the field. As a result, there isn't any single textbook or lecture note that covers all the materials we will be talking about. (In fact, the instructor is hoping to write high-quality lecture notes that can be used by future students and researchers.) Relevant reading materials will be linked on the course webpage each week; the instructor can provide additional references upon request.

Related Courses

Here is a list of similar/related courses offered by other instructors.

- *Algorithms and Combinatorics for Geometric Graphs* by Luca Castelli Aleardi and Éric Colin de Verdière

- *Algorithms for Planar Graphs and Beyond* by Erik Demaine, Shay Mozes, Christian Sommer, and Siamak Tazari
- *Sparsity* by Marcin Pilipczuk and Michał Pilipczuk
- *Optimization Algorithms for Planar Graphs* by Philip Klein and Shay Mozes
- *One-Dimensional Computational Topology* by Jeff Erickson
- *Graph Minor Theory* by Sergey Norin

Teaching Methods and Expectations

The class will mostly be ***lecture-based*** in a seminar style. The instructor and the students will take turns to present and to scribe. There will also be ***reading assignment*** and ***homework problems*** to complement the lectures. Every student is expected to finish the reading assignment before the class, taking notes and contributing in discussion during the class, and solidify the understanding of the material by solving the homework problems. And we will use the scribe notes and homework submissions to determine if the students are actively engaging in learning.

This course is at graduate-level and fairly fast-paced. We will hand out homework 0 in the first week and give the students an idea of the level of background knowledge expected.

Collaboration Rules

You are *strongly encouraged* to use all the resources online/offline. You are also encouraged to work with other groups (and also people outside the course) and discuss your progress, as long as in the end each group individually writes down the solutions in their own words, and properly cites everyone and everything you have consulted. In particular, if you find a solution manual online that contains an answer to the problem, cite the solution manual. If chatting with your friends at gaming night gives you an idea to solve the problem, cite your friends. If your cat walks across your keyboard and sends you to a Wikipedia page that contains a crucial construction to the proof, cite your cat. However, you don't have to cite the lectures and any course related materials. Please refer to the Academic Honor Principle and Sources and Citations at Dartmouth if you have doubts, or contact The Office of Community Standards & Accountability.

Grading

The final grade will be based on presentations, class participation, homework grades, and potentially an optional *research project*.

The tentative weights are 50% on presentation and scribe work, and 50% on homework. If the student choose to do project it will serve as substitute for the homework. Because we are likely to have a small-size class, I would prefer to talk to each of you and set up concrete goals after reviewing your submission to homework 0. The grading will also take into account the difference between student groups (undergrads vs graduates).

Tentative Course Schedule and Assignments

All materials are subject to change based on the actual class interaction and student feedback.

Combinatorial Properties

- ***Rotation System and Duality***
  - tree-cotree decomposition
  - Euler formula, discrete Gauss-Bonnet
  - degree formula
- ***Separators***
  - Lipton-Tarjan tree separator
  - cycle separator
- ***Graph Minors***
  - Kuratowski's Theorem
  - surfaces
  - Treewidth
  - Grid minors
  - Robertson-Seymour structure theorem

Representations

- ***Drawing***
  - Hanani-Tutte
  - crossing lemma
  - Schnyder wood and grid embedding
  - medial curves, electrical and homotopy moves
  - Fary theorem, straight-line drawing
  - strong product theorem, O(1)-queue number
- ***3D Polytope***
  - Steinitz theorem
  - Koebe-Andreev-Thurston disk-packing
  - separator from disk packing
- ***Spectral***
  - Tutte's spring embedding
  - Maxwell-Cremona correspondence: planar frameworks, reciprocal frameworks, polyhedral lifts
  - Separator through small eigenvalue cut
  - Colin de Verdiere invariant

Algorithmic Tools

- ***Distance Compression***
  - r-divisions
  - Klein's multiple-source shortest paths
  - Monge property
  - VC-dimension
  - emulators
- ***Distance Sketching***
  - path separators, eps-covers, and distance oracle
  - KPR low-diameter decomposition
  - bidimensionality
  - low-hop partition
  - cop decomposition

Topological Graph theory

- ***Sparsity***
  - shallow-minors, bounded-expansion
  - polynomial expansion, small separator
- ***Beyond planar graphs***
  - k-planar, quasi-k-planar
  - geometric intersection graphs
- ***Misc topics***
  - Embedding into surfaces, facewidth
  - Separating cycles, low-intersecting system on surfaces
  - Graph lifts and Negami conjecture
  - Discharging methods
  - Borsuk-Ulam Theorem

Academic Honor Principle

The faculty, administration, and students of Dartmouth College acknowledge the responsibility to maintain and perpetuate the principle of academic honor, and recognize that any instance of academic dishonesty is considered a violation of the Academic Honor Principle.

Religious Observances

Dartmouth has a deep commitment to support students' religious observances and diverse faith practices. Some students may wish to take part in religious observances that occur during this academic term. If you have a religious observance that conflicts with your participation in the course, please meet with me as soon as possible—before the end of the second week of the term at the latest—to discuss appropriate course adjustments.

Student Accessibility and Accommodations

Students requesting disability-related accommodations and services for this course are required to register with Student Accessibility Services (SAS; Apply for Services webpage; student.accessibility.services@dartmouth.edu; 1-603-646-9900) and to request that an accommodation email be sent to me in advance of the need for an accommodation. Then, students should schedule a follow-up meeting with me to determine relevant details such as what role SAS or its Testing Center may play in accommodation implementation. This process works best for everyone when completed as early in the quarter as possible. If students have questions about whether they are eligible for accommodations or have concerns about the implementation of their accommodations, they should contact the SAS office. All inquiries and discussions will remain confidential.