Exponential-time hypotheses [IPZ'01]

2/17/21 10:25 AM

Assuming $P \neq NP$ (CNFSAT $\notin$ P), all NP-hard problems can't be solved in poly-time.

No subexp. time algorithms in practice!

COLORING, INDSET, DOMSET, HITTINGSET, HAMPATH ...

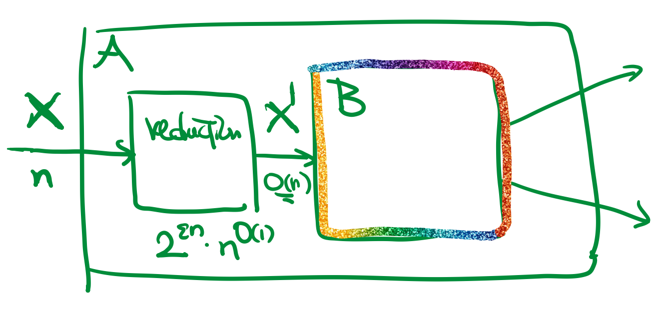
CNFSAT: n variables, m clauses, $\sim \Theta(n)$ [IPZ'01]

k-SAT: k literals per clause.

ETH: CNFSAT can't be solved in $2^{o(n)}$ time.
k-SAT " $2^{\delta_k \cdot n}$ for some δ_k.

Subexp-time reduction.

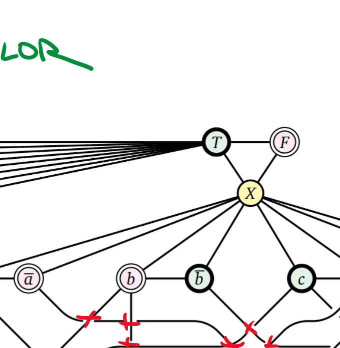
- subexp. time ($2^{\varepsilon n} \cdot \text{poly } n$ $\forall \varepsilon > 0$) during reduction.
- linear blowup to input size.



CNFSAT
$A \leq B$
$n \mapsto n^2 = N$
2^n $\quad 2^{\sqrt{N}}$

example. MAXINDSET

G:



Φ: $(a \vee b \vee c) \wedge (b \vee \bar{c} \vee \bar{d}) \wedge (\bar{a} \vee c \vee d) \wedge (a \vee \bar{b} \vee \bar{d})$

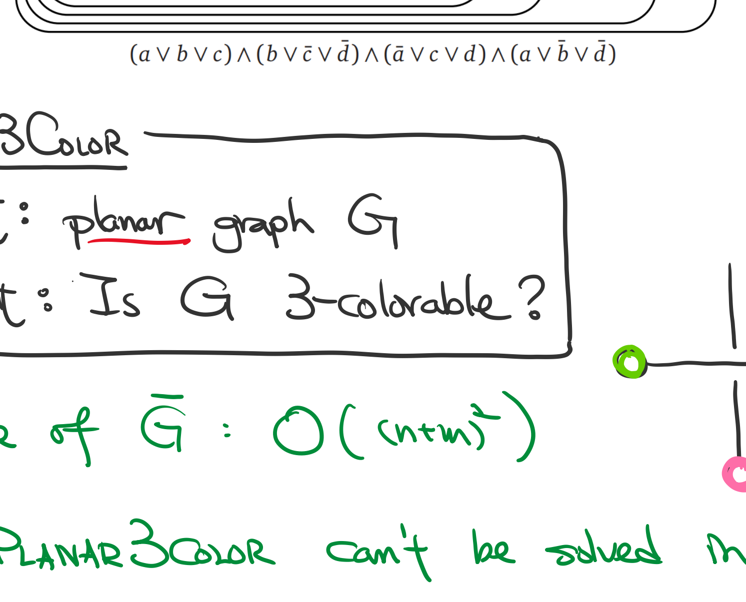
m: #clauses.

3SAT $\leq$ MAXINDSET
$\Phi \mapsto (G, m)$
$O(n+m)$ $\quad O(n+m)$
size of G:
literals in Φ
$O(n+m)$

Cor. MAXINDSET not in $2^{o(n)}$ time assuming ETH.

example. 3COLOR

3SAT $\leq$ 3COLOR.



$(a \vee b \vee c) \wedge (b \vee \bar{c} \vee \bar{d}) \wedge (\bar{a} \vee c \vee d) \wedge (a \vee \bar{b} \vee \bar{d})$

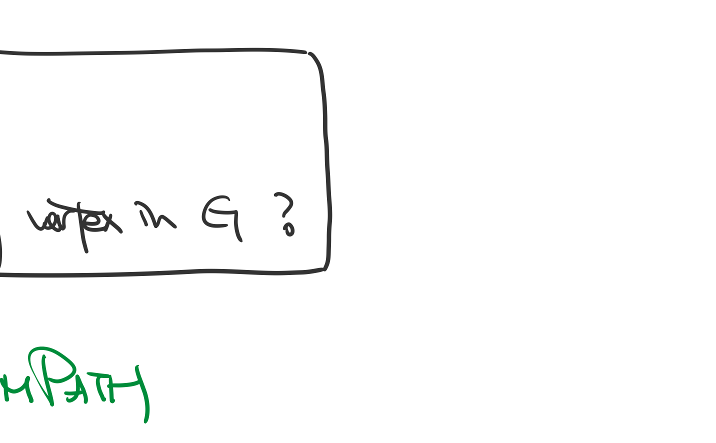
size of G:
2 · # variable $O(n)$
O(1) · #clause $O(m)$
$O(1)$

$O(n+m)$
$\sim O(n)$

Cor. 3COLOR can't be solved in $2^{o(n)}$ time assuming ETH.

PLANAR 3COLOR

input: planar graph G output: Is G 3-colorable?



size of $\bar{G}$: $O((n+m)^2)$

Cor. PLANAR 3COLOR can't be solved in $2^{o(\sqrt{n})}$ time.

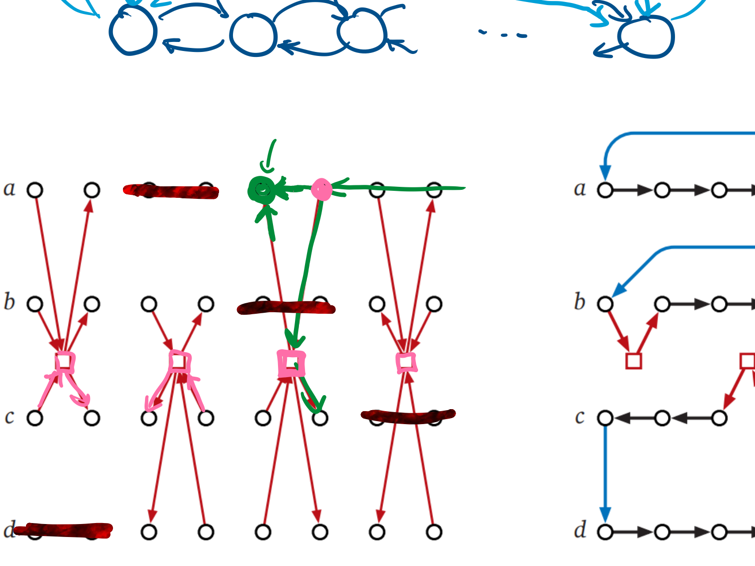
HAMPATH

input: directed graph G. output: Is the path through every vertex in G?

3SAT $\leq$ HAMPATH

pf sketch. $\Phi \Rightarrow G$.

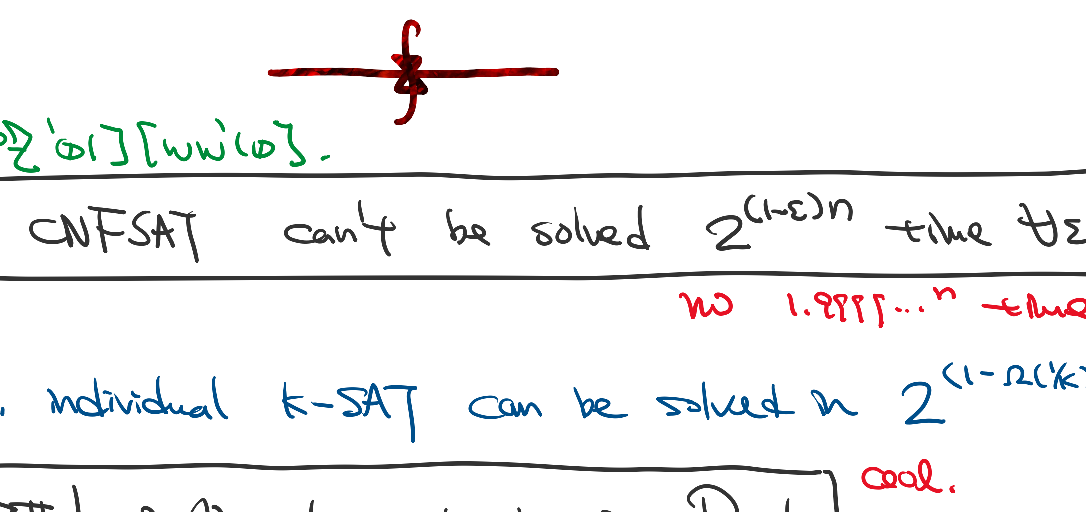
variable:



size of G:
$m \cdot n$
$n \mapsto n^2$
Cor. No $2^{o(n)}$ ~~$2^{o(\sqrt{n})}$~~ -time algorithm.

clause:

≤ 3 appearances per variable.
$\Downarrow$
size $O(n+m)$.
$(|\hat{G}| = O(n+m))$



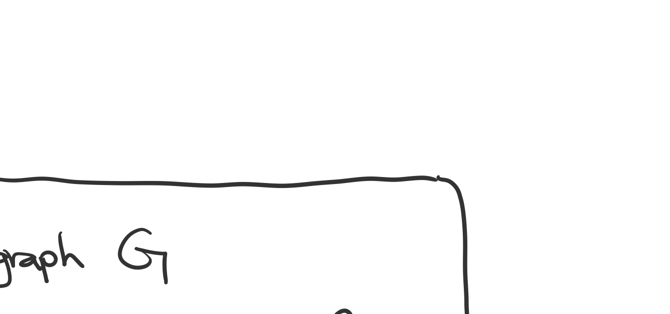
[W'05] [IPZ'01] [WW'10].

SETH: CNFSAT can't be solved $2^{(1-\varepsilon)n}$ time $\forall \varepsilon > 0$.

no $1.9999...^n$ time.

- careful. individual k-SAT can be solved in $2^{(1-\Omega(1/k)) \cdot n}$ time.

SETH implies lower bounds in P! cool.



$n \mapsto 2^{n/k} = N$

$2^{n \cdot \frac{k-\varepsilon}{k}} = 2^{(n/k) \cdot (k-\varepsilon)}$ $\quad N^{k-\varepsilon}$

example.

DIAMETER

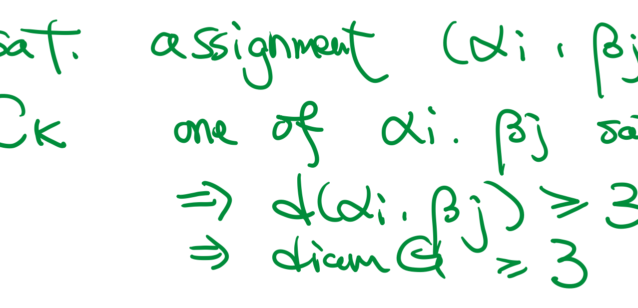
input: graph G output: Is diam(G) ≥ 3?

CNFSAT $\leq$ DIAM.
$\Phi \mapsto G$
Φ sat. $\quad$ diam(G) ≥ 3
not sat. $\quad$ " $= 2$

Build G: var. $x_1, \ldots, x_{n/2}$ $\quad x_{n/2+1}, \ldots, x_n$
α_1 $\quad 0, \ldots, 0$ $\quad 0, \ldots, 0$ $\quad \beta_1$
$\vdots$
$\alpha_{2^{n/2}}$ $\quad 1, \ldots, 1$ $\quad 1, \ldots, 1$ $\quad \beta_{2^{n/2}}$

$2^{n/2}$ $\quad C$ $\quad 2^{n/2}$

clauses:



size of G:
V: $O(2^{n/2} + n)$
E:

$\alpha_i - C_k$ iff α_i does not sat. C_k
$(0, 0 \ldots)$ $\quad (x_1 \vee x_2 \vee \bar{x}_n)$

- Φ sat.: $\exists$ sat. assignment (α_i, β_j)
  $\forall C_k$ one of α_i, β_j sat. C_k.
  $\Rightarrow d(\alpha_i, \beta_j) \geq 3$
  $\Rightarrow$ diam(G) ≥ 3
- Φ unsat.: $\forall$ assignment (α_i, β_j)
  $\exists C_k$ s.t. both α_i, β_j not. sat C_k.
  $\Rightarrow d(\alpha_i, \beta_j) = 2$

How about $d(C_k, C_{k'})$? add clique.
$\Rightarrow$ diam(G) $= 2$. ☒.

- LCS/Edit dist. [BK'15] [ABW'15]
- k-Dominating set. [W'05]
- BCP [W'05]

$\vdots$

No consensus on SETH.

SETH true (no fast CNFSAT) $\Rightarrow$ P lower bounds.

SETH false (faster CNFSAT) $\Rightarrow$ circuit lower bounds.

P vs NP of our generation.