NP-hardness reductions

2/15/21 10:44 AM

To prove Problem A is NP-hard, reduce known NP-hard problem to A

3SAT ≤P

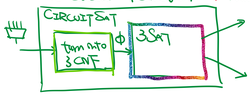
Input: 3-CNF formula where each literal occurs twice.
Output: Is the formula satisfiable?

3-SAT NP-hard. Complete.

$(a \vee b \vee c) \wedge (b \vee \bar{c} \vee \bar{d}) \wedge (\bar{a} \vee c \vee d) \wedge (a \vee \bar{b})$

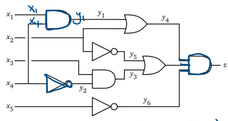
CIRCUIT-SAT ≤P 3SAT.

arbitrary
1. circuit ⇒ 3-CNF formula. specific, restricted.
2. yes inst. ⇒ yes inst.
3. no " ⇒ no "
4. check reduction (1) runs in poly-time.



mapping reduction

pf. (sketch)



$(y_1 = x_1 \wedge x_4) \wedge (y_2 = \overline{x_4}) \wedge \dots \wedge (\quad)$ $(x_1) \wedge (x_2) \wedge \dots \wedge (x_3)$

$(x_1 \vee \bar{x}_2) \wedge (x_2 \vee \bar{x}_3) \wedge \dots \wedge (x_k \vee \bar{x}_1)$.

$a = b \wedge c \longmapsto (a \vee \bar{b} \vee \bar{c}) \wedge (\bar{a} \vee b) \wedge (\bar{a} \vee c)$

$a = b \vee c \longmapsto (\bar{a} \vee b \vee c) \wedge (a \vee \bar{b}) \wedge (a \vee \bar{c})$

$a = \bar{b} \longmapsto (a \vee b) \wedge (\bar{a} \vee \bar{b})$

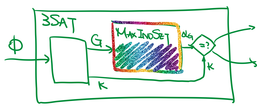
MAX-IND-SET

Input: graph G, int k
Output: ~~largest~~ ∃ ind. set in G of size ≥ k.

has degree ≤ 4

3SAT ≤P MAX-IND-SET.

3-CNF formula ⇒ (graph G, int k.)

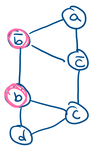


pf sketch. 1. $\phi \Rightarrow (G, k)$.

Build G: clause gadget:

$(a \vee \bar{b} \vee \bar{c}) \Rightarrow$ True

constraint/conflict gadget: $(b \vee c \vee d)$



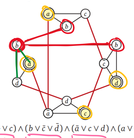
#vertices in G: 3 · #clauses in ϕ.

Set k: k = #clauses in ϕ.

2. ϕ sat. ⇒ G has ind. set of size k.

3. ϕ unsat. ⇒ G has no "

4. all in poly-time



$(a \vee b \vee c) \wedge (b \vee \bar{c} \vee \bar{d}) \wedge (\bar{a} \vee c \vee d) \wedge (a \vee \bar{b} \vee d)$

a = T, c = T, d = F, b = whatever.

MAX-IND-SET S, G — MAX-CLIQUE S, $\bar{G}$ — MAX-VERTEX-COVER $\bar{S}$, G

3COLOR

Input: graph G
Output: Is G 3-colorable?

3SAT ≤P 3COLOR

pf sketch. 1. $\phi \Rightarrow G$

Build G:

- variable:

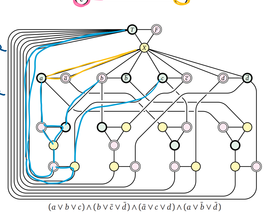
- Truth gadget.

- clause:

2. ϕ sat. ⇒ G 3-colorable

3. ϕ sat. ⇐ G 3-colorable

4. all in poly-time.



$(a \vee b \vee c) \wedge (b \vee \bar{c} \vee \bar{d}) \wedge (\bar{a} \vee c \vee d) \wedge (a \vee \bar{b} \vee d)$