Regular languages are automatic: NFAs

1/18/21 12:53 PM

Administrivia:

- HW1 due on Friday (1/22)
  - Watch out for deadly sins!
- Office hours: Mon, Tue 4p-5p. Thu 1p-2p

Regular: representable by reg. expressions.

Automatic: accepting by DFAs.

Let's try to prove our first nontrivial result:

All regular languages are automatic.

⇒ Construct DFA for reg. expression.

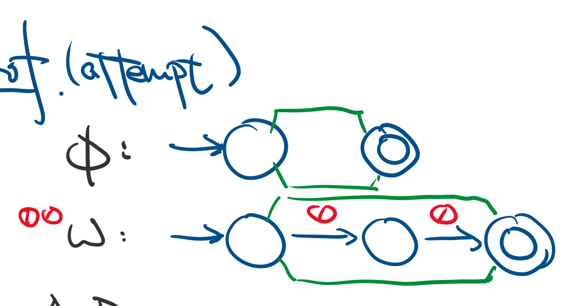
Reg. expression:

$\emptyset$	$\emptyset$	} construct DFAs.
w	$\{w\}$ for any $w \in \Sigma^*$	
$A+B$	$L(A) \cup L(B)$	} combine DFAs.
AB	$L(A) \bullet L(B)$	
A^*	$L(A)^*$	

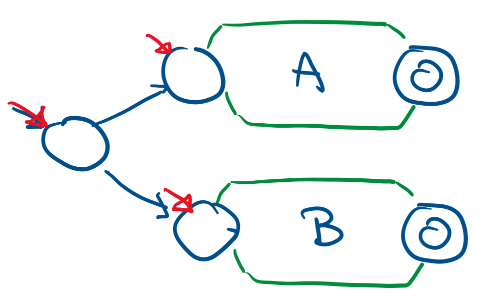
Proof (attempt)

$\emptyset$:

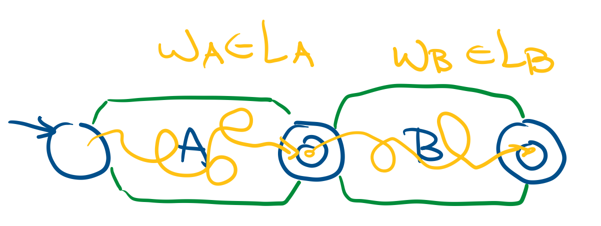
w:



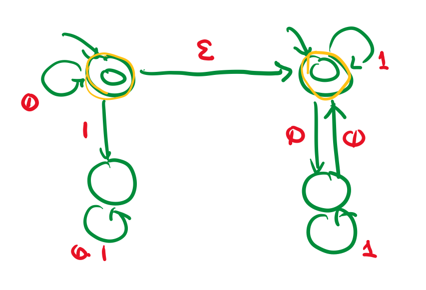
A+B: "run M_A, M_B in parallel".



A•B:



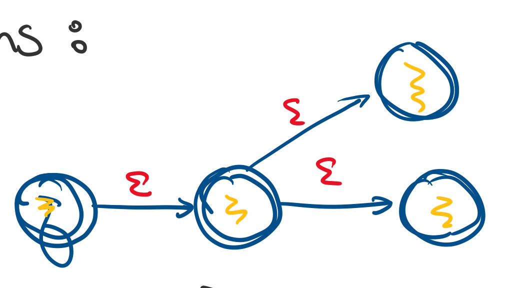
example: $L_A = 0^*$ $L_B = \{w : \text{even } \#0s\}$



- What if we augment DFA w/ ε-transitions?

Finite Automata w/ ε-Transitions:

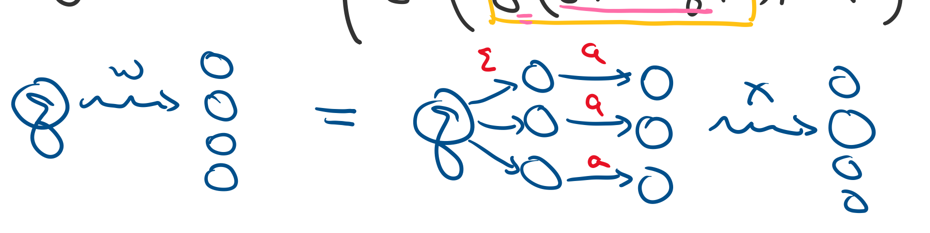
- Q, s, A
- $\Sigma_\varepsilon := \Sigma \cup \{\varepsilon\}$



$\varepsilon\text{-Reach}(q) := \{r \in Q : q \overset{\varepsilon}{\rightsquigarrow} r\}$

- $\delta : Q \times \Sigma_\varepsilon \to 2^Q := \{\text{all subsets of } Q\}$

$$\delta^*(q, w) := \begin{cases} \varepsilon\text{-Reach}(q) & \text{if } w = \varepsilon \\ \delta^*(\delta(\varepsilon\text{-Reach}(q), a), x) & \text{if } w = a \cdot x,\ a \in \Sigma,\ x \in \Sigma^* \end{cases}$$

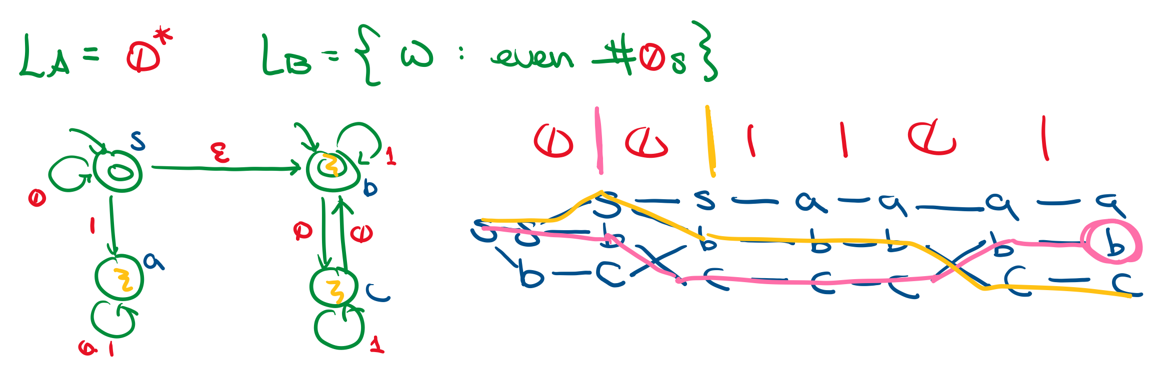


$$\left(\delta(S, a) := \bigcup_{q \in S} \delta(q, a), \quad \delta^*(S, w) := \bigcup_{q \in S} \delta^*(q, w) \right)$$

M **accepts** w if $\delta^*(s, w)$ contains some accepting states. ($\cap A \neq \emptyset$)

"One of me was successful."

example: $L_A = 0^*$ $L_B = \{w : \text{even } \#0s\}$



Proof. regular ⇒ ε-automatic (accepted by DFA+ε)

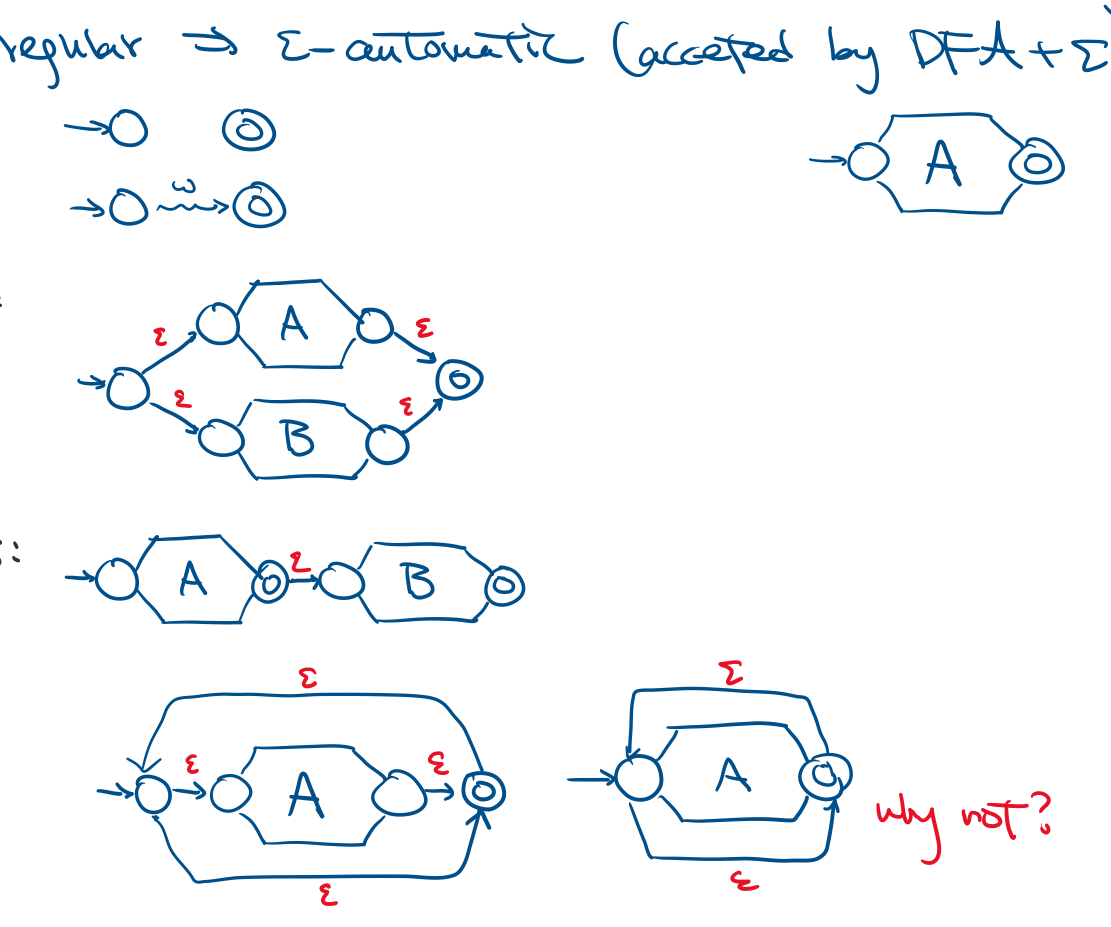
$\emptyset$:

w:

A+B:

A•B:

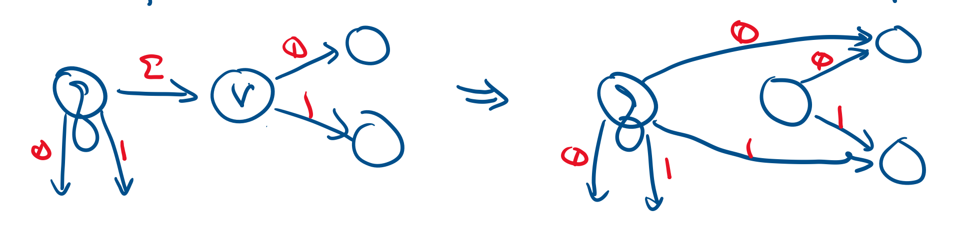
A^*

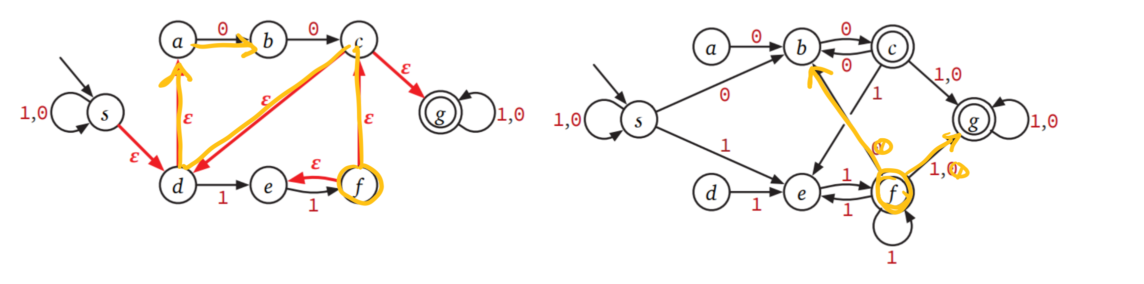


Thm. Every regular language is ε-automatic.

R.E. → DFA → DFA+ε

We can drop the ε-transitions, w/ a cost:





Nondeterministic Finite Automata (NFA).

- Q.
- S multiple starting states
- A multiple accepting states
- Σ_ε w/ ε-Transition
- $\delta : 2^Q \times \Sigma_\varepsilon \to 2^Q$.

definitions are not sacred.

$$\delta^*(P, w) := \begin{cases} \varepsilon\text{-Reach}(P) & \text{if } w = \varepsilon \\ \delta^*(\delta(\varepsilon\text{-Reach}(P), a), x) & \text{if } w = ax \end{cases}$$
