- A problem *cannot be solved in* $O(2^{\delta n})$ time if for all functions $f(n) \in O(2^{\delta n})$, there exists an input of sufficiently large size n for which the algorithm requires time $> f(n)$.
- If the **Exponential Time Hypothesis (ETH)** is true, then *no algorithm can solve* CNF-SAT *in time* $O(2^{\delta n})$ *for some* $\delta > 0$.
- If the **Strong Exponential Time Hypothesis (SETH)** is true, then *no algorithm can solve* CNF-SAT *in time* $O(2^{\delta n})$ *for any* $0 < \delta \leq 1$.

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1. We are given a system of m linear inequalities on n variables. *All coefficients A_{ij} and B_j are integers*:

$$\begin{cases} A_{11}x_1 + A_{12}x_2 + \cdots + A_{1n}x_n & \leq & b_1 \\ A_{21}x_1 + A_{22}x_2 + \cdots + A_{2n}x_n & \leq & b_2 \\ \vdots & & \vdots \\ A_{m1}x_1 + A_{m2}x_2 + \cdots + A_{mn}x_n & \leq & b_m \end{cases}$$

   The ***0-1 integer-programming problem*** (01-IP) asks: "Is there a vector $(x_1, ..., x_n) \in \{0,1\}^n$ satisfying all inequalities?"

   (a) Reduce from 3-SAT to show that 01-IP is NP-hard.

   (b) Assuming ETH is true, show that no algorithm can solve 01-IP in time $O(2^{o(\sqrt{mn})})$.

2. In a graph $G = (V, E)$, a ***dominating set*** is set of vertices $S \subseteq V$ so that $N(S) = V$. The DOMINATING-SET problem asks whether a graph G has a dominating set of size k.

   (a) Is DOMINATING-SET in P?

   (b) Reduce an instance ϕ of CNF-SAT with n variables and m clauses to an instance $\langle G, k\rangle$ of DOMINATING-SET with $|V| = k2^{n/k} + m$ vertices.

   (c) How fast can an algorithm be in solving DOMINATING-SET if SETH is true?

3. **If there's time:** Let A and B be two sets of n d-dimensional $\{0,1\}$-vectors. For instance, if $n = 3$ and $d = 4$, set A could be $\{(1,0,0,0),(1,0,1,0),(0,0,0,1)\}$.

   The ***orthogonal vectors*** problem (OV) asks whether there are vectors $\alpha \in A$ and $\beta \in B$ such that α and β are orthogonal:

$$\langle \alpha, \beta \rangle = \sum_{i=1}^{d} \alpha_i \beta_i = 0$$

   (a) Assuming SETH is true, reduce from CNF-SAT to show that no algorithm can solve OV in $O(n^{2-\varepsilon})$ time for any $\varepsilon > 0$.