Let G be an (either undirected or directed) graph. A ***Hamiltonian path*** in G is a path that visits every single vertex in G exactly once. A ***Hamiltonian cycle*** in G is a closed Hamiltonian path that starts and ends at the same vertex. Consider the following problems:

DRTHAMILTONIANPATH
- ***Input:*** *A directed graph G*
- ***Output:*** *Is there a Hamiltonian path in G?*

HAMILTONIANPATH
- ***Input:*** *An undirected graph G*
- ***Output:*** *Is there a Hamiltonian path in G?*

HAMILTONIANCYCLE
- ***Input:*** *An undirected graph G*
- ***Output:*** *Is there a Hamiltonian cycle in G?*

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1. Describe a polynomial-time reduction from HAMILTONIANPATH to DRTHAMILTONIANPATH.

2. Describe a polynomial-time reduction from DRTHAMILTONIANPATH to HAMILTONIANPATH.

3. Describe a polynomial-time reduction from HAMILTONIANPATH to HAMILTONIANCYCLE.

4. Describe a polynomial-time reduction from HAMILTONIANCYCLE to HAMILTONIANPATH. *[Hint: Might be easier to describe an oracle reduction.]*

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5. ***After class:*** Read the proof from Erickson's note that HAMILTONIANPATH is NP-complete.

You may assume the following problems are NP-hard:

CIRCUITSAT: Given a boolean circuit, are there any input values that make the circuit output TRUE?

3SAT: Given a boolean formula in conjunctive normal form, with exactly three literals per clause, does the formula have a satisfying assignment?

MAXINDEPENDENTSET: Given an undirected graph G, what is the size of the largest subset of vertices in G that have no edges among them?

MAXCLIQUE: Given an undirected graph G, what is the size of the largest complete subgraph of G?

MINVERTEXCOVER: Given an undirected graph G, what is the size of the smallest subset of vertices that touch every edge in G?

3COLOR: Given an undirected graph G, can its vertices be colored with three colors, so that every edge touches vertices with two different colors?

HAMILTONIANPATH: Given an undirected graph G, is there a path in G that visits every vertex exactly once?

HAMILTONIANCYCLE: Given an undirected graph G, is there a cycle in G that visits every vertex exactly once?

DIRECTEDHAMILTONIANCYCLE: Given an directed graph G, is there a directed cycle in G that visits every vertex exactly once?

TRAVELINGSALESMAN: Given a graph G (either directed or undirected) with weighted edges, what is the minimum total weight of any Hamiltonian path/cycle in G?