

Find a fooling set of infinite size for each of the following languages, thus showing that all of them are *non-regular*:

1. $L_1 = \{ww \mid w \in \{0,1\}^*\}$
2. $L_2 = \{0^m 1^n \mid n \text{ divides } m \text{ and } m \geq 0, n > 0\}$
3. $L_3 = \{0^m 1^n 0^{m+n} \mid m, n \geq 0\}$
4. Let $\Sigma = \{\rightarrow, \leftarrow, \uparrow, \downarrow\}$. Starting at position $(0,0)$, the strings in Σ^* represent *walks* on an infinite grid. A walk is *closed* if both its starting and ending point are at $(0,0)$. For example, the string $\rightarrow\uparrow\leftarrow\uparrow$ corresponds to a walk starting at $(0,0)$ and ending at $(0,2)$, and string $\uparrow\uparrow\downarrow\downarrow\leftarrow\rightarrow\leftarrow\rightarrow$ corresponds to a closed walk.
Let L_4 be the set of all strings that are closed walks on the infinite grid.
5. L_5 consists of all strings in which the substrings 00 and 11 appear the same number of times.