1. Turn the following regular expressions into equivalent *DFA*s.

   (a) $(01 + 10)^*(0 + 1 + \varepsilon)$

   (b) $1^* + 10^* + 100^*$

   (c) $\varepsilon + 0(0 + 1)^* + 1(0 + 1)^*$

2. Let $N$ be a given NFA $(Q, S, A, \Sigma, \delta)$. The language accepted by $N$ is defined as

$$L(N) \coloneqq \left\{ w \in \Sigma^* : \delta^*(S, w) \cap A \neq \varnothing \right\}.$$

   We showed in class that the language $L(N)$ must be regular/automatic.

   Prove that the following language associated to the given NFA $N$ is also regular/automatic.

$$L^{\forall}(N) \coloneqq \left\{ w \in \Sigma^* : \delta^*(S, w) \subseteq A \right\}$$

3. Let $L$ be an arbitrary regular/automatic language. Prove that the following languages are also regular/automatic.

   (a) $\mathit{complement}(L) \coloneqq \left\{ w \in \Sigma^* : w \notin L \right\}$

   (b) $\mathit{reverse}(L) \coloneqq \left\{ w^R \in \Sigma^* : w \in L \right\}$

   (c) $\mathit{both}(L, L') \coloneqq \left\{ w \in \Sigma^* : w \in L \cap L' \right\}$

   (d) $\mathit{prefix}(L) \coloneqq \left\{ x \in \Sigma^* : xy \in L \text{ for some } y \in \Sigma^* \right\}$

   (e) $\mathit{cycle}(L) \coloneqq \left\{ xy : yx \in L \text{ for some } x,\, y \in \Sigma^* \right\}$