1. ***No fast automata checkers.*** Consider the following problem about NFAs.

   NFA-REJECT
   - ***Input:*** *An n-state NFA N.*
   - ***Output:*** *Does NFA N reject at least one string? In other words, is $L(N) \neq \Sigma^*$?*

   Prove that the NFA-REJECT problem is NP-complete.

   [This demonstrates that, analyzing the behavior of a (succinctly written) program in general is really hard, even when the program has no access to memory!]

2. ***A friend of a friend is my friend.*** Modern social networks model the relationship between people as *graphs*. We want to find a social subgroup within the network such that every person in the subgroup knows each other.

   k-CLIQUE
   - ***Input:*** *An undirected graph G.*
   - ***Output:*** *Is there a clique (complete subgraph) of size k in G?*

   (a) Prove that the k-CLIQUE problem cannot be solved efficiently under the exponential-time hypothesis. What is the best lower bound you can get?

   (For full credit, your lower bound has to imply that the $(\log n)$-clique problem cannot be solved in polynomial time.)

   *(b) Either design an $O(n^{k-\varepsilon})$-time algorithm for the k-CLIQUE problem for some $\varepsilon > 0$, or prove that the problem cannot be solved in $O(n^{k-\varepsilon})$ time for any $\varepsilon > 0$ under the strong exponential-time hypothesis.

   [Hint: How fast can you find a triangle in a graph?]