1. ***Ice-sliding puzzle.*** Every ’90s kid will immediately recall the traumatic experience they had when looking at the picture below.

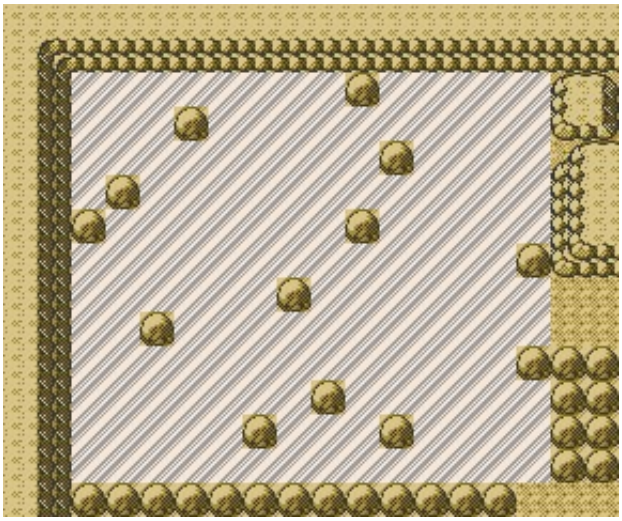

Figure 1. The ice-sliding puzzle in the Ice Path from Pokémon Gold/Silver. The entrance is at the bottom-right and the exit is on the right.

An ***ice-sliding puzzle*** is an $n \times n$ grid, where each cell contains either nothing but *ice*, or occupied by an *obstacle*. Whenever the main character enters the (cell containing) ice from one of the four possible directions, the character will continue to slide straight across unless bumped into an obstacle or a bounding wall. The player has no control when the character is sliding on ice, and can only move again (by pressing one of the four directional keys) when the character is still. The goal of the puzzle is to enter the ice from an *entrance* and leave the ice at the *exit(s)* (both of which can be anywhere on the grid).

One can make the puzzle *even harder* by assuming that there are *items* on the map to be collected. Assume for simplicity that the cells where the items are at are *not* covered by ice, and thus a character sliding across will stop at the exact same spot where the item is.

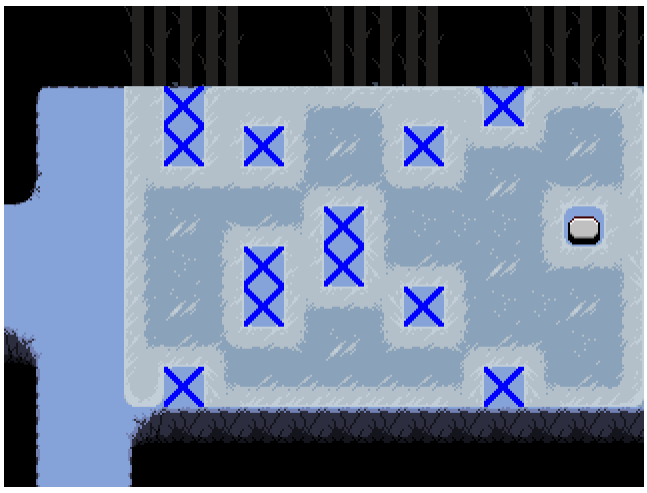

Figure 2. The ice-sliding puzzle from Undertale. The entrance is any spot on the left and the exit is the button on the right. The goal is to stop at every ’X’ symbol and turn them into ’O’.

Let ICESLIDING denote the following problem:

ICESLIDING
- ***Input:*** *The $n \times n$ grid together with the positions of all the obstacles, items, entrances, and exits, as well as an integer k.*
- ***Output:*** *Is there a way to slide across the ice from an entrance to an exit while collecting all the items, using at most k steps/key-presses?*

(a) Prove that ICESLIDING is in NP.

(b) Prove that we can find a sequence of at most k key presses solving the puzzle in polynomial time given an oracle to ICESLIDING.

(c) Either prove that ICESLIDING is NP-hard, or solve ICESLIDING in polynomial time.