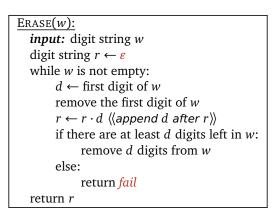
Really!? Prove (formally) that the language of the regular expression 0\*1(10\*1+01\*0)\*10\* corresponding precisely to the set of positive integers divisible by 3, represented in binary (potentially with leading 0s).

[Hint. Separate your proof into two directions. Construct a DFA for one direction and use induction to prove the other direction. What are the induction statements? You might need to strengthen the induction hypothesis.]

2. *Erasing digit sequence.* Let the input be a string of digits from 0 to 9 (in other words, the alphabet set  $\Sigma$  is  $\{0, \ldots, 9\}$ ). The ERASE function is defined as follows:



A digit string *w* is *erasable* if ERASE(*w*) successfully returns another digit string. For example, string w = 314159265358979323846264338327950288419 is erasable because

ERASE(w) = 314159265358979323846264338327950288419 = 355243251.

Construct DFAs that recognize the following languages.

- (a)  $\{w \in \Sigma^* : w \text{ is erasable}\}$
- (b)  $\{w \in \Sigma^* : \text{both } w \text{ and } \text{ERASE}(w) \text{ are erasable} \}$

[It is not sufficient to just draw the diagram; you must explain your construction, especially what each state represents, in English. (This is equivalent to commenting your code with the meaning of each variable.) Remember your job is to convince the reader that your construction is correct. Alternatively, you may describe the DFAs using the formal tuple ( $Q, s, A, \Sigma, \delta$ ). But you still need to explain your construction. Answers without English explanations will receive no credit, even when the answers are correct.]