1. ***Really!?*** Prove (formally) that the language of the regular expression $\mathtt{0}^*\mathtt{1}(\mathtt{10}^*\mathtt{1}+\mathtt{01}^*\mathtt{0})^*\mathtt{10}^*$ corresponding precisely to the set of positive integers divisible by 3, represented in binary (potentially with leading $\mathtt{0}$s).

   [Hint. Separate your proof into two directions. Construct a DFA for one direction and use induction to prove the other direction. What are the induction statements? You might need to strengthen the induction hypothesis.]

2. ***Erasing digit sequence.*** Let the input be a string of digits from $\mathtt{0}$ to $\mathtt{9}$ (in other words, the alphabet set Σ is $\{\mathtt{0}, \ldots, \mathtt{9}\}$). The ERASE function is defined as follows:

```
ERASE(w):
  input: digit string w
  digit string r ← ε
  while w is not empty:
      d ← first digit of w
      remove the first digit of w
      r ← r · d   ⟨⟨append d after r⟩⟩
      if there are at least d digits left in w:
          remove d digits from w
      else:
          return fail
  return r
```

   A digit string w is ***erasable*** if ERASE(w) successfully returns another digit string. For example, string $w = \mathtt{314159265358979323846264338327950288419}$ is erasable because

   ERASE(w) = **3**~~141~~**5**~~92653~~**5**~~89793~~**2**~~38~~**4**~~6264~~**3**~~383~~**2**~~79~~**5**~~02884~~**1**~~9~~ $= \mathtt{355243251}$.

   Construct DFAs that recognize the following languages.

   (a) $\{w \in \Sigma^* : w \text{ is erasable}\}$

   (b) $\{w \in \Sigma^* : \text{both } w \text{ and } \textsc{Erase}(w) \text{ are erasable}\}$

   [It is not *sufficient to just draw the diagram; you* must *explain your construction, especially what each state represents, in English. (This is equivalent to commenting your code with the meaning of each variable.) Remember your job is to* convince *the reader that your construction is correct. Alternatively, you may describe the DFAs using the formal tuple $(Q, s, A, \Sigma, \delta)$. But you still need to explain your construction. Answers without English explanations will receive no credit, even when the answers are correct.]*