- You know the drill now: Find students around you to form a ***small group***; use ***all resources*** to help to solve the problems; ***discuss*** your idea with other group member and ***write down*** your own solutions; raise your hand and pull the ***course staffs*** to help.
- This worksheet is *optional*; ***no submissions required***, although we encourage to try it out and test your understanding.

Our topic for this working session is on *fingerprinting*.

To check if two objects A and B are equal, first choose another object r at random, and compare the *sketched* objects A_r and B_r somehow, so that if $A_r = B_r$ then $A = B$ with high probability. Choosing the right random object to combine with is where the creativity lies!

In class we showcase two possibilities. Let $A[1:n]$ be an array given; assume the value stored in each $A[i]$ is either 0 or 1.

- The fingerprinting by taking modulo over random prime number can be viewed as choosing a fixed number 2 and a random prime number p, set $x = 2$ and compute

$$A(2) = \sum_i A[i] \cdot 2^i \pmod{p}$$

- The fingerprinting by (Demillo-Lipton-)Schwartz-Zippel lemma can be viewed as choosing a random number a and a fixed prime number p, set $x = a$ and compute

$$A(x) = \sum_i A[i] \cdot x^i \pmod{p}$$

Solve the following problems using the technique of fingerprinting.

1. Let A and B be two *sets*, each of size n. Verify whether $A = B$ by comparing at most $O(\log n)$ bits with good probability.

2. Let A and B be two *multisets*, each of size n. Verify whether $A = B$ by comparing at most $O(\log n)$ bits with good probability. *[Hint: First try to do it in polynomial time, regardless of the fingerprinting size.]*

To think about later: (No submissions needed)

3. Let A and B be two *binary trees*, each of size n. Verify whether $A = B$ in polynomial time with good probability.

4. Let A and B, and C be three $n \times n$ matrices. Verify whether $AB = C$ in $O(n^2)$ time with good probability.