- You know the drill now: Find students around you to form a ***small group***; use ***all resources*** to help to solve the problems; ***discuss*** your idea with other group member and ***write down*** your own solutions; raise your hand and pull the ***course staffs*** to help; ***submit*** your writeup through Gradescope in *24 hours*.

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Our topic for this working session is on *NP-hardness reductions*.

Let's recall the fundamental concept of polynomial-time reducibility. There are two variants:

- A problem $L$ is ***oracle reducible*** to a problem $R$ when we can solve any instance of $L$ in polynomial time by querying an oracle of $R$ that correctly solves any instance of $R$.
- A problem $L$ is ***mapping reducible*** to a problem $R$ if there is a polynomial-time algorithm $f$ such that given any input $w \in \Sigma^*$, we can produce an output $f(w)$, such that $w \in L$ if and only if $f(w) \in R$.

Today we will focus on mapping reductions. Ingrained the following idea in your mind:

> To show a problem $R$ is NP-hard, find another NP-hard problem $L$ and (mapping) reduce $L$ to $R$. Construct an instance of $R$ from an *arbitrary* instance of $L$ in polynomial time, such that yes/no-instance of $L$ corresponds to yes/no-instance of $R$ respectively.

In class we have demonstrated three examples (3SAT, MAXINDEPENDENTSET, 3COLORING); it is time for you to construct your first NP-hardness reduction.

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Let $G$ be an (either undirected or directed) graph. A ***Hamiltonian path*** in $G$ is a path that visits every single vertex in $G$ exactly once. Consider the following problems:

DirectedHamPath
- ***Input:*** *A directed graph $G$*
- ***Output:*** *Is there a Hamiltonian path in $G$?*

HamPath
- ***Input:*** *An undirected graph $G$*
- ***Output:*** *Is there a Hamiltonian path in $G$?*

1. Describe a polynomial-time reduction from HamPath to DirectedHamPath.
2. Describe a polynomial-time reduction from DirectedHamPath to HamPath.

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*To think about later: (No submissions needed)*
A ***Hamiltonian cycle*** in $G$ is a closed Hamiltonian path that starts and ends at the same vertex.

HamCycle
- ***Input:*** *An undirected graph $G$*
- ***Output:*** *Is there a Hamiltonian cycle in $G$?*

3. Describe a polynomial-time reduction from HamPath to HamCycle.
4. Describe a polynomial-time reduction from HamCycle to HamPath. *[Hint: Might be easier to describe an oracle reduction first.]*
5. Read the proof from Erickson's note (Section 12.11) that DirectedHamCycle is NP-hard. Modify the proof to show that DirectedHamPath is NP-hard. (This implies all three problem we worked with today are NP-hard.)

*Conceptual question: Exponential-time hypothesis (ETH)* is a stronger assumption than P≠NP, stating that 3Sat cannot be solved in subexponential time (that is, $2^{o(n)}$). If we reduce 3Sat to problem $P$, can we say that $P$ cannot be solved in subexponential time either under ETH?