Finite automata, automatic languages

Friday, April 4, 2025 11:44 AM

Administrivia.

- Remember to submit worksheet to Gradescope (label the Qs!)
- HW0 due next Wednesday (Apr 9)

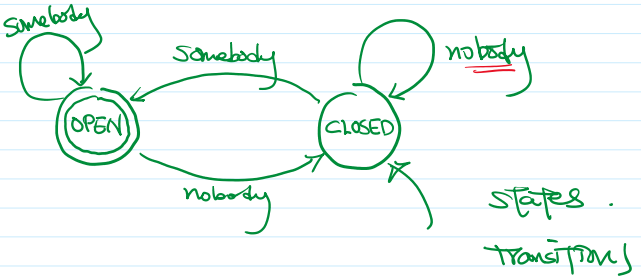
Question. What essential components do we need for a computer?

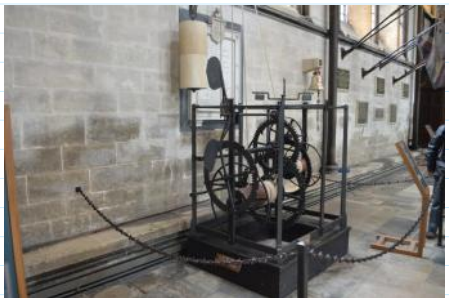
CPU, memory (RAM, ROM), input/output,

For sake of univ. computation, let's go for simplicity!

CPU

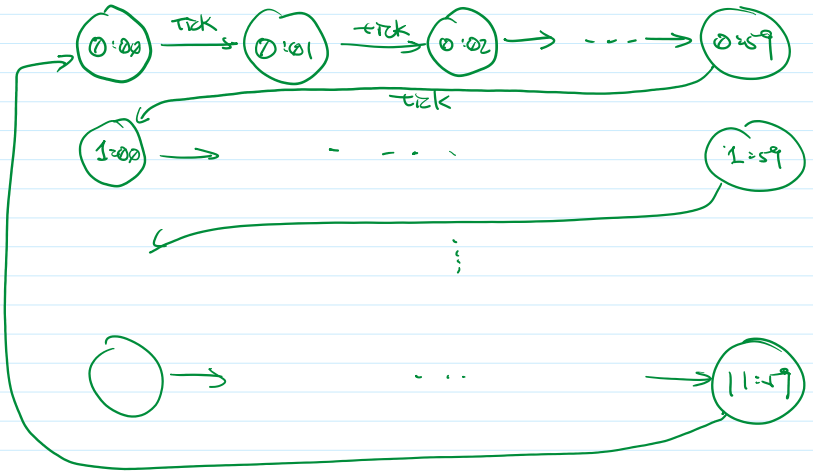
Look at other designs...?





Salisbury Cathedral Mech. Clock [1386]

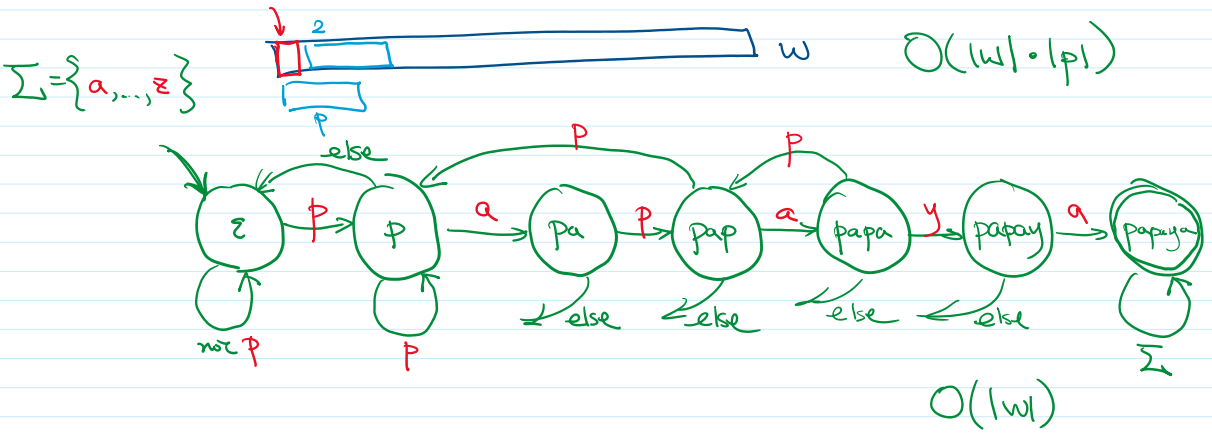
The Orloj [1410]



String matching automata (KMP) [Knuth-Morris-Pratt 1970]

StringMatching (w , p) : (p = papaya)

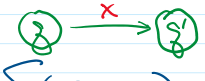
Decide if w contains p as substring



Deterministic Finite Automata (DFAs) $(Q, s, A, \Sigma, \delta)$

All these set/elements have to be finite!

States:	Q
Start state:	$s \in Q$
Accepting states:	$A \subseteq Q$
Input alphabet:	Σ
Transition function:	$\delta : Q \times \Sigma \to Q$ (Curr state, next, next)



$\delta(q, x) = q'$

Δ δ ∂ d

Input alphabet: Σ

Transition function: $\delta : Q \times \Sigma \to Q$ (Q: curr state, Σ: next symbol, Q: next state) $\quad \delta(q, x) = q'$

Extended transition fun: $\delta^* : Q \times \Sigma^* \to Q$

$$\delta^*(q, w) = \begin{cases} \delta^*(\delta(q, x), w') & \text{if } w = x \cdot w' \\ q & \text{if } w = \varepsilon \end{cases}$$

($\delta(q, x) = q'$)

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ACCEPT?(M, w):
  Input: DFA M = (Q, s, A, Σ, δ)
  Output: does M accept w?
  q ← s
  for i ← 1 to |w|:
      q ← δ(q, w[i])
  return [q ∈ A]
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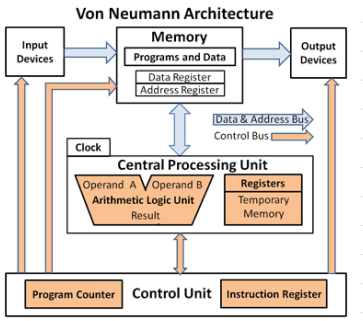
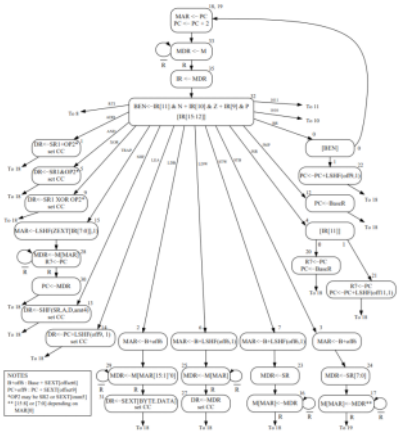
Modeling / Philo.

Question 1. Does DFA really captures CPU...?





Theory

Question 2. What type of languages do DFAs compute?

Def. DFA M accepts w if $\delta^*(s, w) \in A$.

Language of M:

$$L(M) := \{ w \in \Sigma^* : M \text{ accepts } w \}$$

Def. Languages accepted by some DFAs are call automatic.

e.g. $L(KMP_{papaya}) = \{w \in \Sigma^* : w \text{ contains papaya as substring}\}$

$L_3 := \{w \in \Sigma^* : w \text{ not contain 000 nor 111 as substring}\}$

memory?

$L := \{w : \#0s \text{ x } \#1s \text{ differ by} \leq 2 \text{ in every prefix of } w\}$

Question 3. DFA seems to be better than reg. exp.
Are all regular languages automatic?

Problem	Language
Progamming language	Machine Models.
No-memory programs	Regular Expressions

Progamming language	Machine Models.
No-memory programs	Regular Expressions
O(1)-memory programs	DFA.
Algorithm	Instance of a model.