Problems, Languages, and Encoding

Wednesday, April 2, 2025 11:54 AM

Administrivia.

- HW0 is out. due next Wed.
- Add yourself to Canvas / Gradescope / Slack.
- Stop me if you have questions!

Def. A problem has input and output.

- We need a language / symbol set to talk about it!

PRIME? (int n)
YES if n is prime.

Problem.

Def. Alphabet Σ, character $x \in \Sigma$

$\Sigma := \{0, 1\}$ $\Sigma' :=$ English alphabet.

Word / String $\Sigma^k = \overbrace{\Sigma \times \Sigma \times \Sigma \times \cdots \times \Sigma}^{k}$

$01001 \in \Sigma^5$

string $\in \Sigma'^6$ string is a string (over Σ')

Empty word $\varepsilon \in \Sigma^0$

$A \sqcup B$: $A \cup B$ × $A \times B$ are disjoint.

Concatenation xy

define

Set of all strings. $\Sigma^* := \{\varepsilon\} \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \cdots$

Def. A language is a subset of Σ^*.

Intuition: language is the formal realization of a decision problem.

HASZERO?
input: binary string w
output: does w have 0?

Decision problem P.
$P : \Sigma^* \to \{\text{yes}, \text{no}\}$

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Language of HASZERO?

$\Sigma := \{0, 1\}$

$L := \{0, 01, 10, 00, 000, 001, \ldots\}$

SORTED?

input: int. array $A[1..n]$.
output: is A sorted?

$w\varepsilon = w$

Language of SORTED?

$\Sigma := \{0, 1, \ldots, 9, [,], ,\}$

$L = \{[n_1, n_2, \cdots n_k] : n_i \in \{0,\ldots,9\}^*, n_i \le n_{i+1} \ \forall i\}$

$A = [0, 2, 7, 11] \xrightarrow{\text{encoding}} [0, 2, 7, 11]$

(Decision) Problem —encoding→ Language

Def. Language of a decision problem P w.r.t. some encoding

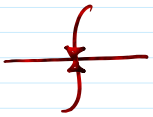
$\{w \in \Sigma^* : P(w) = yes\} \subseteq \Sigma^*$

input: $\forall n$ set of integers $\{x_1, \ldots, x_n\}$.
output: is $x_1^2 + \ldots + x_n^2 < 0$?

$L = \phi = \{\}$

input: $\forall n$ set of integers $\{x_1, \ldots, x_n\}$.
output: is $x_1^2 + \ldots + x_n^2 = 0$?

$L = \{\{0\}, \{0,0\} \ldots\}$.
$\{0\}^* = \{\varepsilon, 0, 00, 000 \ldots\}$



Question. What can we solve without any knowledge?
by if-then, while-loop, sequencing?

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Def. Let L_A, L_B be languages.

Union (If-then) $L_A \cup L_B := \{ w \in \Sigma^* : w \in L_A \text{ or } w \in L_B \}$

Concatenation (Seq.) $L_A \circ L_B := \{ w \in \Sigma^* : w = xy, \ x \in L_A \times y \in L_B \}$

Kleene-Star (while-loop) $L_A^* := \{\varepsilon\} \cup L_A \cup L_A \circ L_A \cup L_A \circ L_A \circ L_A \cup \cdots$
$= \{ w : w = x_1 x_2 \cdots x_k, \ \forall i \ x_i \in L_A \}$

Def. A language L (over Σ) is **regular** if L equals:

- ϕ — output no
- $\{w\}$ — output yes when input is w
- $L_A \cup L_B$ — L_A, L_B regular
- $L_A \circ L_B$
- L_A^*

Is w? (word x)
if ($x = w$) output Yes

Def. Regular Expression : $\phi, w, A \cup B, AB, A^*$.

$0 1^*$ $\quad (01)^*$ $\quad 1(0+1)^* = 1(0^*1^*)^*$ $\quad (0^*1^*)(0^*1^*)$
$0 \cdot 1^*$ $\quad \{01\}^*$ $\quad 010$ $\quad \emptyset \ 1 \ \emptyset \ \varepsilon$

- All strings containing 000 as substring
- All strings .. subsequence
- All strings not containing 000 as substring.

Question. What are the class of languages decided by RE.?
What problems can we solve without brains?