- The homework is due on April 9, 23:59pm. Please submit your solutions to Gradescope.
- You should submit Homework 0 *individually*. Please write down the names of the members *very clearly* on the first page of your solutions.

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1. ***Neighborhoods in graphs.*** Let G be an undirected graph with *at most* one edge between any pair of vertices, but possibly with self-loops (an edge whose head and tail is the same vertex). Define V and E to be the vertex set and edge set of G, respectively.

   The ***neighborhood*** function $N : V \to 2^V$ takes a vertex v and maps it to the subset of vertices adjacent to v in G. We also define the neighborhood of any *subset* of vertices S as the union of each individual neighborhoods, one for each vertex in S. In notation,

   $$N(S) := \bigcup_{v \in S} N(v).$$

   Fix a starting vertex s in V. The ***k-th neighborhood*** of s is defined to be $N(N(\cdots N(s)\cdots))$, where $N(\cdot)$ is applied k times.

   (a) Describe an algorithm to decide if the following is true: for every vertex t in V, there is an integer k such that the k-th neighborhood of s contains t.

   *(b) Describe an algorithm to decide if the following is true: there is an integer k, such that for every vertex t in V the k-th neighborhood of s contains t.

2. ***Balanced parentheses.*** ***Balanced parentheses*** is a string over the two symbols [and], defined *recursively* as one of the following:

   - an empty string ε;
   - string [w] for some balanced parenthesis w;
   - string xy for some *nonempty* balanced parentheses x and y.

   For example, [[[][]]][][[[]]][] is a balanced parentheses string of length 18.

   (a) Prove by induction that removing any pair of consecutive symbols [] (if exists) from any balanced parentheses results in another balanced parentheses.

   *(b) Prove by induction that removing any pair of consecutive symbols][(if exists) from any balanced parentheses results in another balanced parentheses.