Discrete Mathematics in Computer Science

COSC 30/ENGS 66 (Spring 2024)

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Time and Place

Monday, Wednesday, Friday 2:10pm – 3:15pm, Thursday 1:20pm – 2:10pm (X-hour)
ECSC, Room 116

Course Description

This course covers the mathematical foundations of computer science—arising in many algorithms, concepts, and techniques—that are not calculus-based. We will begin by introducing the concept of a *mathematical proof*, following by various proof techniques and the formal mathematical language required, through a selection of introductory topics in discrete mathematics such as graph theory and combinatorics (counting). Guiding examples from computer science will motivate the topics studied.

Course Objectives

At the end of the course, the students are expected to be able to:

- Reason with formal mathematical notations, knowing when to use them and when not to
- Construct convincing and rigorous mathematical arguments
- Achieve better clarity in thinking that enables one to learn, create, and communicate ideas
- Dissect bigger task/problem into smaller ones through clear-cut interface
- Correctly observe recursive structures in problems and apply induction
- Model problems at hand using graphs when appropriate
- Manipulate discrete objects and arithmetic expressions at will

Prerequisite

Math 3 (Calculus) and *COSC 10 (Problem Solving)* are required as prerequisites. *COSC 10* may be replaced with *COSC 1 (Introduction to Programming & Computation)* or *ENGS 20 (Introduction to Scientific Computing)* with instructor permission.

Related Courses

- *COSC 70 Foundations of Applied Computer Science*: This class *complements* COSC 30 and focuses on other mathematical aspects of computer science that are often application-oriented, like linear algebra, probability and statistics, and numerical methods. The students are strongly encouraged to take both classes to build a solid foundation in mathematical reasoning.[1]

[1] You might have heard people said that software engineers don't need to learn advanced mathematics. Well, if you look at the good ones, they are all strong in reasoning and making arguments to back up their design decisions, even if they don't work on projects that are inherently math-heavy.

- *COSC 31 Algorithms*: This is the natural *sequel* of COSC 30. Algorithms are ubiquitous in computer science; this course focuses on the design and analysis of (mostly classical) algorithms, with similar emphasis on the development of rigorous and unambiguous ways of communicating ideas.
- *COSC 39 Models of Computation*: This is the *flip side* of COSC 31. The theory of computation studies the power of computers through the abstract lenses of Turing machines and their variations, and how they relate to the rest of computer science. The ability to abstract and formalize computation itself as a concrete mathematical object, sufficiently robust that encapsulate all the physical computing devices known to human beings, is one of the biggest discoveries in human history.

There are also full-term classes on chosen topics in discrete mathematics like *Math 20 Probability*, *Math 28 Introduction to Combinatorics*, and *MATH 38 Graph Theory*. I am happy to point you to more advanced courses if you find out that theoretical computer science might be your passion.

Textbook

The following textbook will be our main reference:

- *Mathematics for Computer Science* by Lehman, Leighton, and Meyer (LLM), Jun 2018 version.

Relevant reading materials will be linked on the course webpage each week.

It is always good to remember that no textbooks are perfect, and at the modern age there are more online resource than one can possibly consume to learn about a specific topic (and are often times better at engaging the audience). So I encourage you to build your study habits around using multiple resources to learn; after all, we always try to use every possible tools available to solve our problems at hand, and learning is no different.

Teaching Methods and Expectations

Every week, the four scheduled lecture hours (including the X-hour) will be split between *working sessions* and *lectures*.

Lectures. We will start each course unit (each with one central concept) with an one-hour lecture (usually on Wednesdays and Fridays). I firmly believe that the real learning only happens when the students are actively engaging the materials through a set of well-designed exercises, and not through passive learning like sitting in lectures. Consequently, the students are *not* required to attend the lectures live. That being said, I will work hard to make sure the lectures are fun and inspirational to earn your attention. It is also a good time to see a concept at work to ask clarifying questions. We will attempt to record the lectures and upload them after class.

Working sessions. Every course unit will start with a *working session*. This is normally on Mondays and Thursdays. The working sessions are required to participate. The students will split into smaller groups and work on the exercises together. The instructors, including me and the teaching assistants, will be around helping to answer questions and, from time to time, leading the discussion. Each student has to write down their own solutions to submit. Optional practice problems may be provided for self-study.

Office hours. Outside regular lecture hours the instructor and TAs will provide office hours on a weekly basis; all students are strongly encouraged to attend these discussion-based sessions.

Assignments. Assignments come in two formats. First, each week we will point to chapters from the Lehman-Leighton-Meyer book as ***reading assignment*** to complement the lectures. Then there are weekly ***homework problems***. These problems are designed specifically at the right difficulty so that it is challenging (and even sometimes frustrating), but rewarding and aiming to expand our knowledge and skills after solving them (maybe with some help from your peers). Therefore it is okay if you find the problems hard; that is *by design*. The students are expected to spend a reasonable amount of time (measured in hours) working on the assignments. In a sense, these problems are the real driver of the class. You should treat them as the main work you need to finish every week, and use whatever resource to help you (including the lectures, work session, readings, discussion, office hours, and internet).

Some problems are marked with a *star*; these are more challenging (and fun) problems, and are therefore optional and worth *extra credits*. Working on these problems will help you master the materials and skills taught in this class. I strongly encourage you to try some of them if you are thinking about having a future career that assumes sharp thinking skills and math maturity. The extra credit problems do not count towards raw final grade calculations.

Exams. We will have two *midterms* and one *final exam*. These are normally 180-minute closed-book open-cheatsheet written exams. The goal of the exams is to evaluate your capability and understanding of the course material, based on the Course Objectives.

Collaboration Rules

For homeworks, you are allowed to work in groups each up to *two* people. Each group will submit a solution together; the submitted work should be clearly marked with the names of the group members.

You are *strongly encouraged* to use all the resources online/offline toward solving the problems. You are also encouraged to work with other groups (and also people outside the course) and discuss your progress, as long as in the end each group individually writes down and submit their own version of solutions in their own words, and properly cites everyone and everything you have consulted. In particular, if you find a solution manual online that contains an answer to the problem, cite the solution manual. If chatting with your friends at gaming night gives you an idea to solve the problem, cite your friends. If your cat walks across your keyboard and sends you to a Wikipedia page that contains a crucial construction to the proof, cite your cat. However, you don't have to cite the lectures and any course related materials. Please refer to the Academic Honor Principle and Sources and Citations at Dartmouth if you have doubts.

Submission and Lateness Policy

Homeworks will be announced in class and posted to Gradescope. All homeworks should be submitted electronically through Gradescope. There is no strict format requirements about your solutions; the only important thing is to present and communicate your ideas in a clear, correct, and concise fashion. Also make sure the lighting works out if you decide to take pictures of your handwritten solutions. (I recommend you to convert the photos to black and white first.) The graders get to decide if they can read and understand your solutions or not.

No late submissions will be allowed. This is a rather strict policy that exists for various reasons:

- ensure everyone is following the class;
- help the graders to provide timely, fair, and useful feedback;
- reduce administrative overhead in handling extension requests and bookkeeping.

If you have legitimate reasons for extensions, no need to worry! Not all problems will be counted towards your final grades. (See the Grading section.) You are still strongly encouraged to work on all the problems; the policy is in place to prevent any unforeseen situations coming up.

Feedback to Homeworks

Solutions will be provided to all (non-extra-credit) problems, along with their grading rubrics. We will provide written feedback and comments to your answers. Read them carefully and bring your questions (if any) to the office hours. You may submit regrade requests if you think we misunderstood your solutions. A clear explanation to what you think might be the confusion points must be included. We will regrade them based on the following rules.

- Remember that we can only grade the problems based on what you *wrote*, not what you *think*. Analogously, people will only listen to what you *actually* communicate, not what you think you have communicated.
- We will regrade the problem from scratch. In particular, your grade might go *down*.

Grading

There is always a clash between the instructor–student relation and the examiner–examinee relation. As an instructor I try to provide the best-quality deliverables to help with your study, and as an examiner it is my job to provide the most brutally honest assessment and feedback about your understanding of the material. The former is usually collaborative and supportive, whereas the latter often feels more distant and sometimes adversarial. When it comes to grading, the latter aspect gets amplified and can in fact reduce the effectiveness of the learning. I will try my best to be transparent about the course policies and designs, so that you can reason and understand why a rule is set up that way. If you have doubts, ask me a question. Remember, professors are also human beings (although sometimes we don't feel like one).

Final grades. Here is a brief explanation on how we determine the final course grades.

- *Computing the raw final score.* The raw final score is computed based on the following:
  - exercises during working sessions (10%)
  - homework problems (30%)
  - midterms and final (20% each, 60% in total)

  We will drop the *lowest one-fifth* of the problems (rounding up) from the exercises and homework sets. For example, if a total of 25 problems are given throughout the term, only 20 of them will be counted towards the final grades. In addition, there might be *forgiven problems* granted directly by the instructor to handle special situations; those problems are assumed to be non-existed and other homework problems will be assigned with higher weights.

- *Identify outliers.* Anyone who demonstrated unusual skills and understanding to the materials (for example, solving multiple extra credit problems or getting 95%+ adjusted score) is considered an outlier, and will be removed from calculating the curved letter-grade cut-offs. We reserve the right to give Fs to students meeting any of (but not restricted to) the following conditions:
  - adjusted score lower than 40%;
  - submitting less than half of the homeworks;
  - missing exams.
- *Determine the letter grade cut-offs.* We will compute letter grades in two ways.
  - The *fixed* letter grades have the cut-off between B-/B+ be 75%, and 15% for each full letter grade above/below(thus 5% per grade bucket). In particular, 40% is the passing grade and 85% is the threshold for A-.
  - The *curved* letter grades have the cut-off between B/B- be the class average and one standard deviation for each letter grade above/below, after removing the outliers.

  The final letter grade will be the *higher* of the two. The instructor reserves the right to *bump up* your letter grade, but not down.

Tentative Course Schedule and Assignments

All materials are subject to change based on the actual class interaction and student feedback.

Module P: Proofs

- Week 1: ***Language***
  - introduction; what is a proof, making arguments
  - jargon (sets, sequences, strings, graphs, cardinality, functions), pigeonhole principle
- Week 2: ***Statements and Proofs***
  - predicates, quantifiers, and logic
  - various proof techniques
- Week 3: ***Induction Proofs***
  - induction and inductive definitions
  - induction is recursion; recursion is induction

Module G: Graphs

- Week 4: ***Graphs***
  - graphs, degree counting, hand-shaking lemma
  - walks, paths, and cycles; reachability, whatever-first search
- Week 5: ***Trees***
  - trees and their properties

  - recursion tree, tree induction
- Week 6: ***Directed Graphs***
  - dags, strongly-connected components
  - relation, partial order, equivalence

Module C: Combinatorics

- Week 7: ***Counting and Models***
  - permutations, binomial coefficients
  - inclusion-exclusion, combinatorial equivalence
- Week 8: ***Probability***
  - sample space, event, probability
  - conditional probability
- Week 9: ***Probability***
  - random variables and expectation
  - linearity of expectation

Religious Observances

Some students may wish to take part in religious observances that occur during this academic term. If you have a religious observance that conflicts with your participation in the course, please meet with me as soon as possible, or before the end of the second week of the term—at the latest, to discuss appropriate adjustments. Dartmouth has a deep commitment to support students' religious observances and diverse faith practices.

To assist with calendar planning and awareness of our diverse religious and spiritual community, the list of holy days can be found here. The list represents major holy days which may impact campus events in general, as well as student course attendance, exams, Commencement and participation in activities in the coming year. Thank you for your consideration. If you have any questions about these dates or other concerns, please contact Nancy Vogele, Chaplain and Director of the Tucker Center for Spiritual and Ethical Life.

Student Accessibility Services

Students requesting disability-related accommodations and services for this course are required to register with Student Accessibility Services (SAS; Getting Started with SAS webpage; student.accessibility.services@dartmouth.edu; 1-603-646-9900) and to request that an accommodation email be sent to me in advance of the need for an accommodation. Then, students should schedule a follow-up meeting with me to determine relevant details such as what role SAS or its Testing Center may play in accommodation implementation. This process works best for everyone when completed as early in the quarter as possible. If students have questions about whether they are eligible for accommodations or have concerns about the implementation of their accommodations, they should contact the SAS office. All inquiries and discussions will remain confidential.